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The End of Quantum Theory

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INTRODUCTION

Physics represents the fundamental component of the scientific knowledge, both in its purpose – as it studies the processes and phenomena related to elementary particles, up to the celestial mechanic –, and as a landmark for other sciences. One could add here the fact that theories and discoveries in physics have influenced and still influence decisively both the development of the other sciences and the technological progress in almost all fields of activity. This explains the major interest granted to the development of physics in the XXst century, especially the quantum theory, which stirred a major interest both as depth of knowledge and theoretical way of approach. Nevertheless, the field has been marked by theoretical confrontations and concerns, which have not completely been solved. This is why the atomic structure needs a new approach, a thorough re-evaluation and theoretical re-construction, starting from fundamental hypotheses, redefining the laws that govern the field-substance interaction, to numeric simulation modeling, interpreting and defining new research directions in the experimental field.

This book should be regarded as the result of almost three decades of intense theoretical searches, made by the author upon several difficult questions still looking for an answer:

Which is the structure of the material world?

Which are the laws that govern it?

Is the world of atoms determinist structure or undeterminist one?

Is the atomic internal exchange energy a continuous or a discreet one?

Could the theoretical unification of the atomic-level interaction be achieved?

Should the structure of the atomic electron cloud and the nuclear structure be approached as separate physical entities?

Should the dualism wave-particle be revised?

Which is the relation between formalism and the reality it describes?

Could we have the space-time description of dynamic phenomena in nuclear physics?

Could we disregard the principle of causality in describing the atomic systems?

Are the Maxwell equations fulfilling the theoretical requirements of describing the atomic structure?

Is the unified field theory possible?

We have to specify that the present book represents only the starting point for a new scientific approach of the atomic structure, profoundly different from the quantum description, which tries to answer some of the above-mentioned questions related to the subject in discussion and for many other problems.

Renouncing to the quantum description of the atomic structure has been a long and difficult scientific endeavor, with numerous dilemmas and difficult moments.

The founders of the quantum theory of substance's structure have focused, in their theoretical approach, on the elaboration of the theory that would explain the structure of the material world in the context of insufficient development of the scientific knowledge, so that they had not all the theoretical instruments necessary to solve such a complex issue at that particular moment.

The simplest atomic model is the one of the hydrogen atom, composed of an electron spinning around the nucleus named proton. The experiments from that period have led to the conclusion that practically the entire mass of the atom is located in the center of the nucleus, where the electric positive charge is concentrated.

Therefore, at that moment of the quantum physics theory beginning, an enormous pressure was put on the existing of theoretical physics, aiming to elaborate a new atomic model which would harmonize the theoretical system and the new discoveries in the experimental field, especially the determination of the hydrogen's emission specters, which led to the conclusion that the atomic structure must be a well defined one.

For overstepping these major theoretical obstacles in elaborating a new theory, they used approximations and suppositions, thus straying away the model from the reality they were trying to describe. The chosen solution was to find a fundamental theoretical package with many estimations and they developed an extremely complex characteristic **formalism** (by **formalism** we understand the logical-mathematical ensemble of description and logical-mathematical development of a scientific theory), with a pronounced philosophically explicative conceptual component through which they were hoping to solve such a complex scientific issue – the fundamental structure of the substance.

At the beginning of the 20th century, the fundamental conceptualization in physics and in other sciences was rigorously structured, which impelled an unprecedented development of the theoretical knowledge, which in its turn determined an impressive technical and technological progress.

The technological development has determined an unprecedented progress in the experimental field, which could have not been explained by the theoretical structures existent at that moment. The scientific world was not able to scientifically explain the results of the experimental physics on a continuous increase, which emerged and were closely linked to the substance structure. The simplest model of the atom's structure could not have been elaborated respecting the principles of space, time and determinism, because the general theory of the waves was not sufficiently developed and there was no possibility of numerical modeling of such complex theoretical physics problems. Therefore, at that time they were forced to make use of the available instruments and means, simplifications and estimations.

Abandoning the rigor of the scientific knowledge in its classical form, the scientists drawn the attention to the philosophers of different orientations, who for lack of major preoccupations and new themes of thinking, got involved in this field, uncritically approaching a domain in which the scientific knowledge was not able to clarify with its own means the hypotheses promoted by the quantum theory at its incipient stage. Thus, a philosophical trend of sustaining without reserve the quantum ideas was created, which determined a false legitimacy of the quantum theory, embezzling the scientific knowledge from its natural course.

The alienation from the classical spirit, from its rigorous thinking and norms, the conviction that through formalism the insuperable barriers could be overcome, determined many theoreticians to consider that they finally found a miraculous method of cognition, a fact that led to a certain exaltation regarding the quantum model and to a rejection of the traditional theoretical cognition in physics.

At the beginning of the 20th century, physicians, mathematicians and philosophers focused on the exclusively formal development of the knowledge as a single instrument in describing the atomic model.

New suggestions have been forwarded as philosophical suppositions for grounding the new scientific theory, the energy's discreet structure, the probabilistic and non deterministic character of the atomic structure, which were unconditionally and uncritically accepted, indulging in vain hope that a theory with a high level logical mathematical formalization is implicitly valid. The belief that the probabilistic-type quantum mechanics would solve both the atomic-level cognition problems and the ones in other fields of science was undeniable. Many theoreticians believed and still believe in the theoretical scientific valences of the quantum theory, yet after one hundred years since emerged, no miracle took place in the knowledge field. The quantum theory could not offer a theoretical description of the complex atomic structures. The only model is the one of the hydrogen atom.

They intend to repudiate the classical physics, the ideas of space and time localization, the classical determinism and causality.

The continuous-type description is, from the generality point of view, superior to the discreet-type one and is associated to predictable phenomena; thus, if we know the state of a physical system at the moment t_0 and the laws that describe that system, then we can describe the state of the system at the moment t_1. This principle is successfully applied on a large scale in physics of phenomena, where the ideas of classical determinism, the acceptance of space and time as a measure of processes and phenomena description are completely valid. These classical ideas have led, before the emergence of the quantum theory, to an unprecedented development of the scientific cognition in physics and cannot be removed without putting something else in their stead.

On the most general level, the major error in quantum theory was generated by the incorrect interpretation of the relations between formalism as a theoretical instrument and the reality that the theory was supposing to describe. As one cannot describe discreet-type probabilistic phenomena through systems of continuous functions' equations, nor the continuous systems can be described and interpreted as random systems. The type of the mathematical modeling must correspond to the real system it proposes to theoretically describe.

There are theoreticians who render absolute the importance of the mathematical formalism within a theory. The example is the quantum theory in which the accent is put on formalism or even the theory is reduced to its formal part.

They intend to transfer the features of the formalism to the physical reality. Thus, the features of indeterminism and non-causality specific of the chosen formalism must be assigned to the atomic structure.

The lack of scientific rigor could be exemplified through the dualism wave – particle, which from a strictly philosophical point of view could be formally accepted, but not from a scientific point of view, given the fact that within the same scientific structure a phenomenon cannot have mutually excluding approa-ches and interpretations. In the history of human knowledge all dualisms have ended lamentably.

After a century, the quantum mechanics cannot be drawn in the traditional way of thinking. We cannot accept the process of a phenomenon in the absence of its spatial-time definition. We cannot accept the duality as an explicative scientific model of physical phenomena description.

The blind confidence in formalism and the elaboration of the model ignoring the fundamental features of the reality supposed to be described made of the quantum model of the substance structure an inoperative theoretical instrument deprived of predictive valences.

The transfer of formalism's features over the reality supposed to be described has led to the development of a theory that explains and designs an indeterminist, non temporal and confused hypothetical reality.

Of course, nothing is absolutely and definitively established within the multitude of viewpoints regarding the concurrent theories, but for the traditional way of thinking the strangeness of the quantum ideas is so big that it cannot be accurately expressed through language due both to inner contradictions and to the ideas conflict with the traditional way of thinking. Due to the lack of the temporal component, the quantum theory cannot explain the process of the phenomena.

The aim of the scientific knowledge is that of creating and strengthening a theoretical background in order for the processes and phenomena to become predictable. Even until now the quantum mechanics was not able to elaborate the model of an element having more than two particles. After a century of its existence, it did not succeed in describing with the promoted „theoretical revolutionary instruments” but the first element of Mendeleev’s table.

In the theory of the quantum mechanics the description of phenomena seems to lose the individuality and autonomy of the causal portrayal as a mechanism in describing the motion of elementary particles. In its entirety, the quantum theory is irreconcilable with the idea of causality in its ensemble.

The ambiguous character of the atomic structure’s description is due to its confused system of cognition that incorporates the elements of the system without individualizing them, describing them as a diffuse positional conglomerate in which speed, acceleration, space and time disappear and are replaced by the probability of spatial localization. All of these are against our classical concepts and intuitions.

Unfortunately, a **quantum apologetics** was created and promoted by philosophers of science who do not possess a proper understanding of the quantum theory and of physical processes in their deepness, who describe real physical phenomena and consider the quantum theory as the ultimate form of scientific knowledge. On the other part the physicists do not understand the philosophical essence of quantum theory.

The modern epistemology of science, on this punctual matter, has lost its sense and many exaggerations tend to transform into ridicule a profound and actual theme of the scientific knowledge – the atomic structure.

Wishing to put in accord the quantum theory, whose fundamental characteristic is a discreet one, with the wave theory (as a continuous form of energy's manifestation), which quantum physicist could not deny, and to build a discreet theory structure of the substance, they gave birth to a philosophic improvisation: the **dualism wave – particle**.

The dualism annihilates any form of determinism through the fact that two contradictory theories describing a single reality theoretically induce two separate and irreconcilable conclusions. From the point of view of scientific rigor it is certain that the introduction of two different, mutually excluding concepts within the same theoretical system represents the **dissolution of the scientific knowledge**, which brings the quantum theory near the *science fiction*.

The quantum physics is fatally contradictory to our intuition and our principles of approaching and understanding physical processes and phenomena. It ignores the fact that the sterile formalizations and the accumulation of scientific data have no sense and do not become significant for knowledge unless they are based on a set of hypotheses more suitable proper for the field under investigation.

As a fundamental theme of knowledge, the relation between tradition, represented by the classical physics and innovation, represented by the quantum theory, will definitively get the best of tradition in this case.

THE NEW THEORY OF ELECTROMAGNETIC DYNAMIC ATOMIC STRUCTURE

The aim of this book is that one of promoting a new vision of the material world's nature at the level of the atomic structure, for determining a beginning towards a more profound understanding of the universe of the material world.

The quantum physics has elaborated an indeterminist, non-causal and confuse model, without perceiving the physical essence of the interaction of elementary particles in motion and the interdependence of their action.

A new approach of the atomic and sub-atomic structure based on a new interpretation of its theoretical foundations is necessary for explaining the experimental properties and the theoretical discovery of new properties. We propose a **fundamental theoretical and methodological reconstruction of the atomic and molecular structure** based on a dynamic model of **profound electromagnetic interaction**, which considers both the direct action and the **„self-induction"**, which lays at the theoretical basis of the new concept.

The atomic structure should be approached and understood as a complex **unitary structure**, formed by the atomic nucleus and the electrons cover, permanently interacting and mutually inter-conditioning each other. The new model proposes to answer the problems regarding the **stability and instability of the atomic and molecular structures**, as well as the behavior of the atomic and molecular systems within a magnetic or electric field. The **electro-dynamic atomic model** proposes itself to create models of interaction between atoms and to study the influence and effects of the electromagnetic waves upon the atoms and upon complex atomic structures (groups of atoms); to describe the transitory processes which take place within the atomic structure. We intend to explain the phenomena in its determination and physic process.

The cognition of the atomic structure must arrange and center the entire activity of theoretical knowledge within a unitary system of the structure of material world, which would be in accord with the way of approaching and understanding the physics in its classical form.

In the classical physics the possibility of describing physical phenomena, such as the motions of elements, is postulated by framing them in **space and time**. This hypothesis had an extraordinary success and solved the fundamental problems of knowledge. The settlement of precise links of the succession of all natural phenomena, from **cause to effect**, has led to the hypothesis of a universal determinism based on strict temporally causal relations.

It is necessary to rebuild the theory of the substance's structure taking into consideration the continuous structure of the micro-cosmos energy and the discreet structure of the elementary particles. Many of the experimental data indicated the fact that the atoms must have a complex structure. For opening a new stage in physics, chemistry and biology a new way of thinking must be defined, a new proper model for a profound understanding of the substance's structure.

We propose **the electromagnetic dynamic atomic structure (EDAS)** theory of the substance, where the interactions between the elements composing this structure are essential. This structure allows rendering evident all the complex forces acting as an interaction field – substance and the light's absorption and emission phenomena as electromagnetic waves. Therefore, the new theory of substance's structure becomes completely compatible with the light's wave theory; thus the compatibility of the material world within a high-level unitary theoretical system is achieved.

The new theory is an attempt to build a new „atomic model“ formed of elementary particles moving within the coordinates of time and space, based on the classical way of thinking, which renders evident both the stable states and the transition phenomena described in a classical manner respecting the principles of space and time, the principles of causality and determinism.

We had to find answers and solutions to a big number of scientific and epistemological problems for elaborating a theory with major implications upon the current scientific knowledge. The major difficulty was the dispute over the quantum description of the hydrogen atom, which enjoyed and still enjoys a high reputation in the philosophical and scientific field. Unfortunately, its acceptance is not a validity criterion for theory.

The most difficult moment in over-passing the psychological blockage in elaborating a fundamental theory, different from the quantum theory, was the profound understanding of the significance of philosophical concepts this theory operates with and their delimitation from the scientific foundation of the problem to solve.

In our theoretical approach we considered that it is more important to substantiate and elaborate a new theory in prejudice of a sterile criticism.

We propose to explain the structure of the substance in a classical manner using the cognizance describing the motion of the elementary particles and all the forces acting over them within a spatial and temporal framework. The atom should be regarded as a dynamic structure within which the motion is continuous and the interaction determined by the motion of the elementary particles is fundamental in defining its structure. The ways of the elementary particles interaction with each other are essential landmarks in understanding and elaborating the complex atomic structures.

For being able to describe the atomic structure we developed, within chapter 2, the formalism of the electromagnetic interaction of two bodies with electric charge including into the system the self-induction of the particles' own motion.

We elaborated the theoretical framework described in chapter 1, which from a formal point of view respects the rigors of classical physics and represents the theoretical foundation with a view to the intended theoretical development.

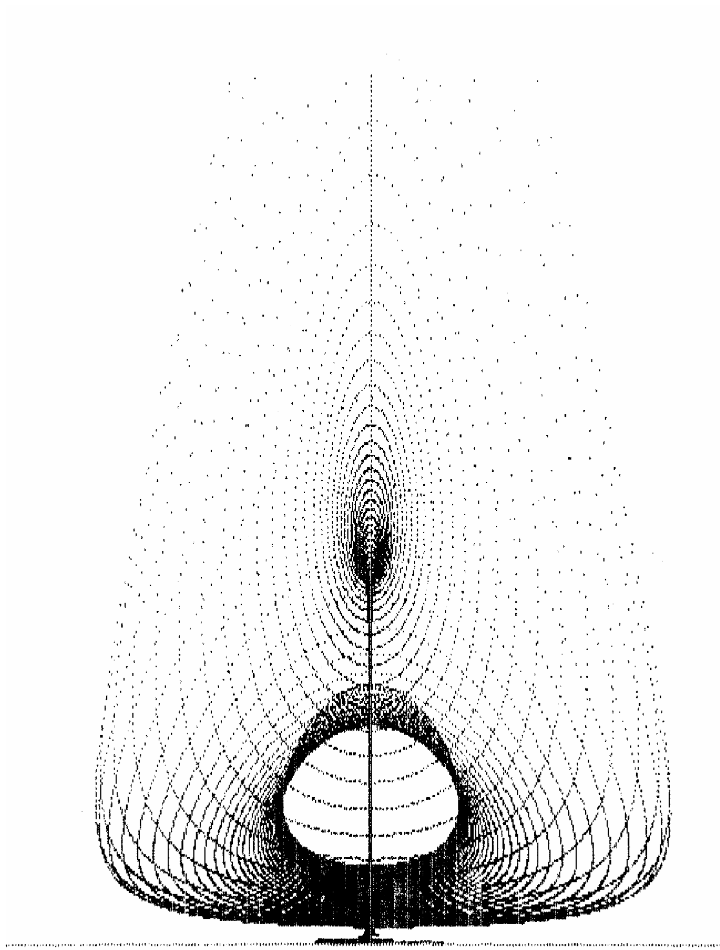
In chapter two we developed a system of 2^{nd} order partial differential equations of hyperbolic type that describes the motion of two bodies with electric charge within a non-inertial system of reference, an exclusive interaction within a closed system in the presence of the electric and magnetic field, of the self-induced field and of its perturbation. The resulted mathematical model represents the solution of the interaction's value for the distance covered by the two particles between two points in a given period of time, which represents a step of interaction. The values calculated after a step of interaction become initial values for the described system. By resuming the numerical calculus we get the description of the dynamic of elementary particles witch are presented in figures 1 to 9.

As follows, we will present the results of the numerical modeling resulted from chapter 2. We specify from the beginning that the graphs represent standardized values of the presented measures and they should be estimated and understood accordingly.

One should notice that the system of equations presented in chapters 2, which describe the interaction of the elementary particles, are not symmetrical equations. Nevertheless, as a result of numerical modeling, one could observe that the dynamic of the elementary particles has a symmetrical character.

In figure 1 we present the projections on the coordinate axes of the electron evolving action in the two plans x,y.

Fig. 1



In figure 2 we present the same projections on coordinate axes x, y but using different input values for the system in use.

Fig. 2

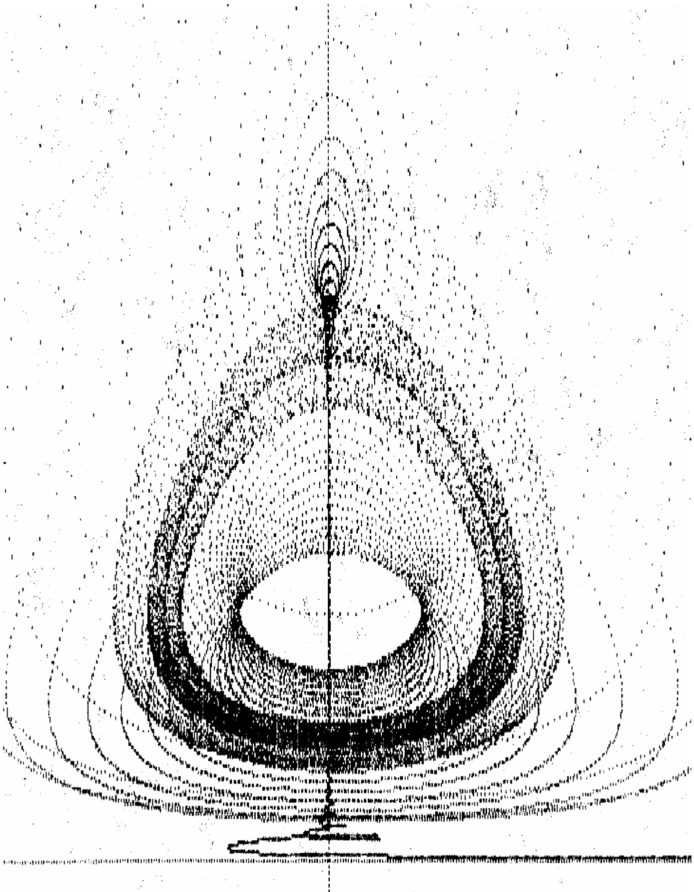
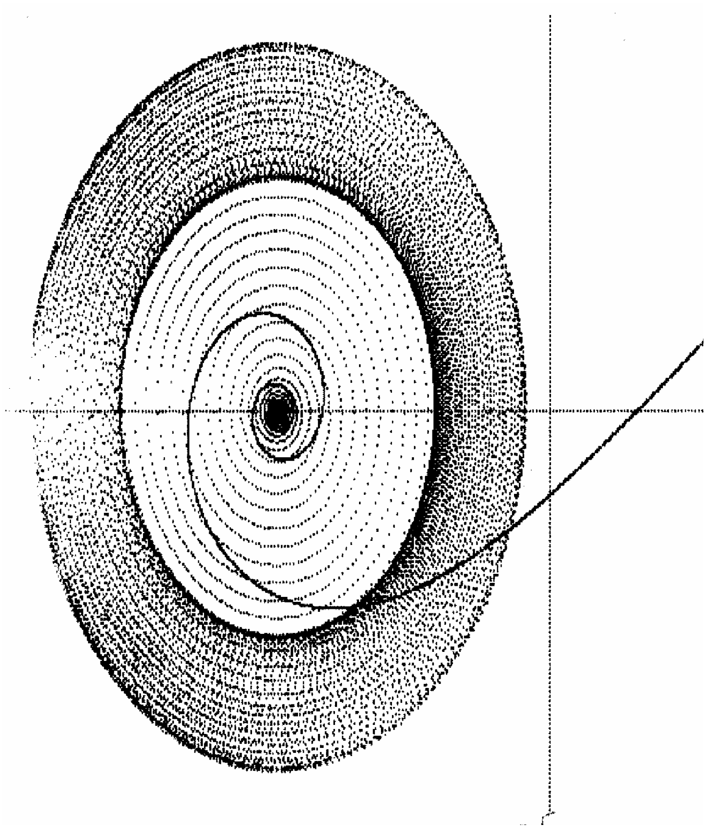
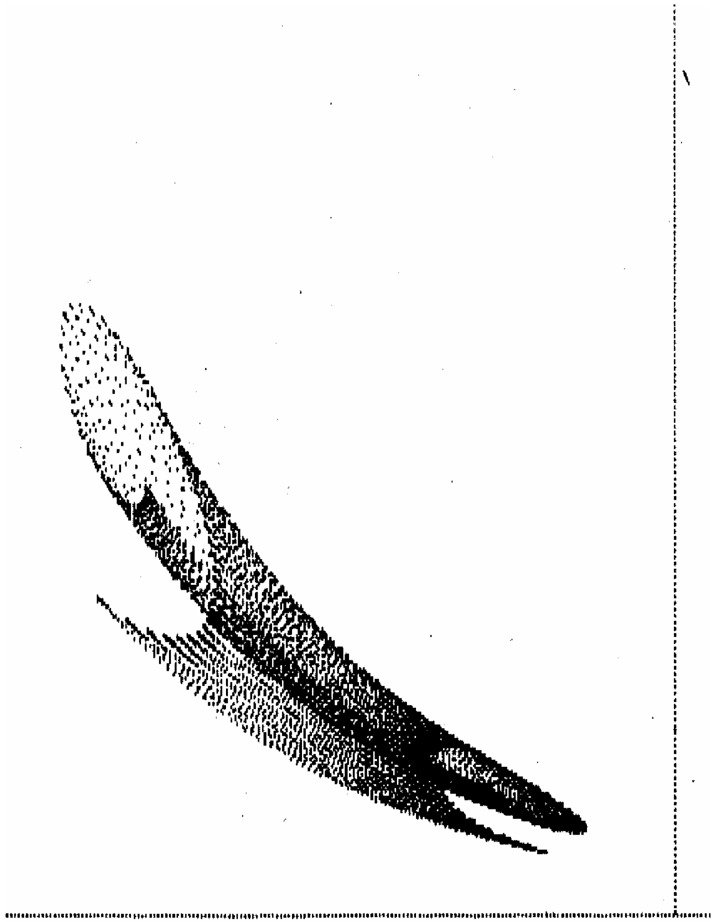


Fig 3



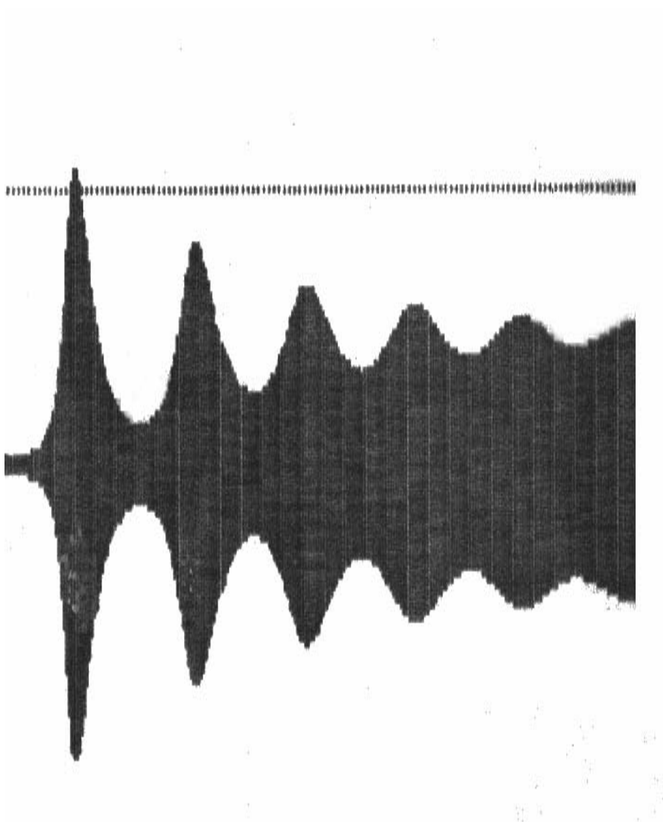
In fig. 3 we present the projections on the coordinate axes of the electron evolving action in the two planes x,z.

Fig.4



In fig. 4 we present the projections on the coordinate axes of the electron evolving action in the two planes y,z.

Fig. 5



In fig. 5 we present the electron evolution amplitude on the coordinate x in time.

All the numerical simulation leads to a stabile trajectory that are presented in figure 7, 8 and 9.

Fig.7

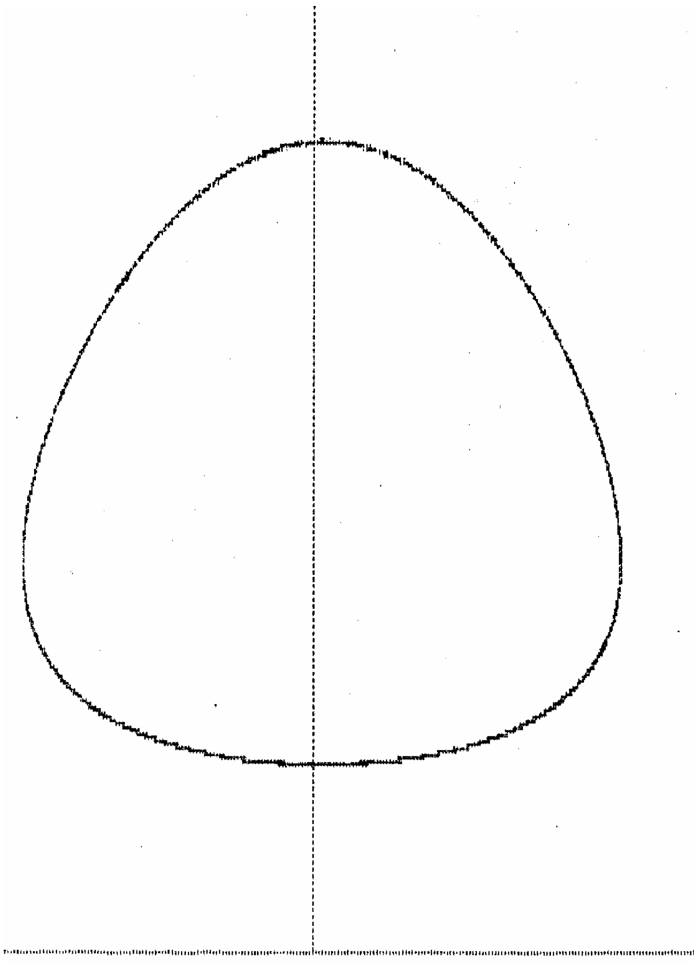


Fig.8

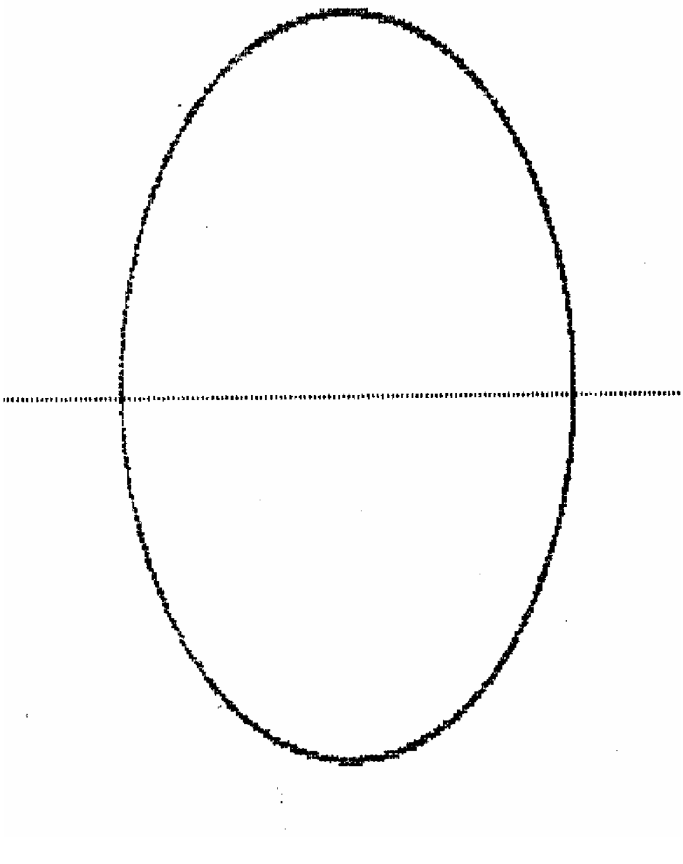
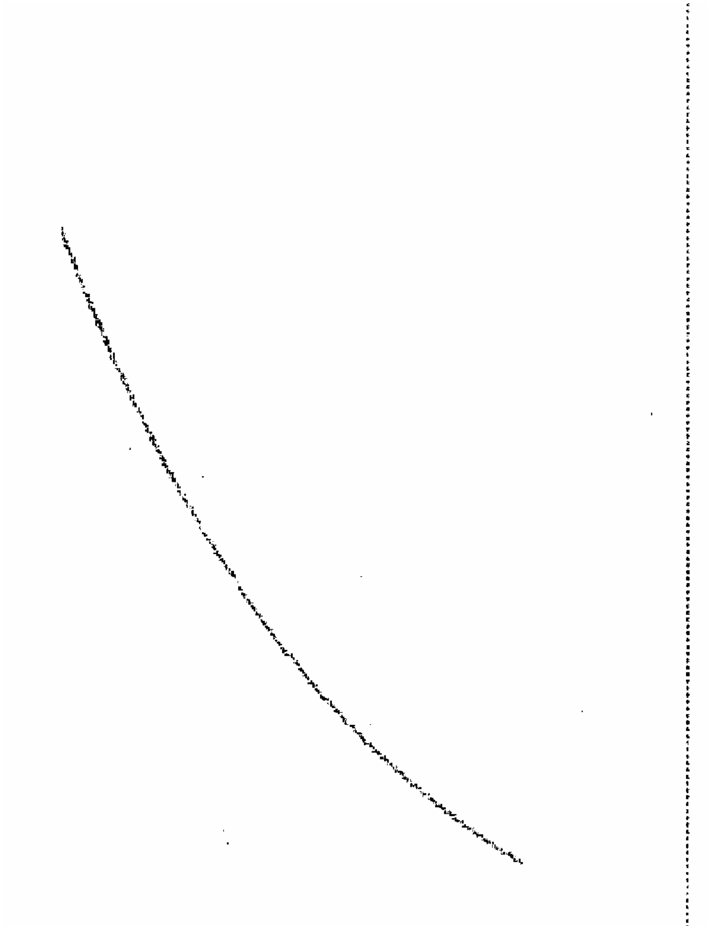


Fig. 9



The formalism that describes the interaction of the dipole could be accepted as a foundation of the interaction of elementary particles at atomic level. **The structural dipole** must be regarded as a whole, a dipole-type complex interaction, as we describe it in chapter 3, among all the elementary particles that compose it.

Through numerical modeling we obtained the graphic representations of a first model of the structure of hydrogen atom, as it is presented in fig. 1 to 9.

The resulted model presents the approximate structure of the complex electromagnetic interaction of two elementary particles in accordance with the principles of the classical determinism, of causality and space - time representation of the atomic structure.

The model allows the calculus at every step of the interaction of all the fundamental measures that characterize the system: position towards the system of coordinates, acceleration, speed, impulse, acting force, oscillation frequency in space-time system. Thus we demonstrate that from a theoretical point of view the motion of an elementary particle could be described in a space-time system, knowing at every moment the position towards the chosen system of reference, the forces acting over the particle, acceleration, impulse and speed of displacement.

Having in view that in chapter 2 the terms of the equation of interaction were implicitly presented within the system of calculus, we developed in chapter 3 the equation of the generalized waves in a system of 2^{nd} order partial equations of hyperbolic type for explaining the theoretical generalization of the electromagnetic interaction for the chosen model from a formal point of view.

The final equations in chapter 2 is the **generalization of the Maxwell equations**, which consists in deductively adding a new non-dissipative term within the equation, o reaction component of the interaction field - substance. This active component of electromagnetic nature, which we called **vector of self-induced**

interaction of the **structural dipole,** represents the component of distortion of dipole's interaction, different from zero. The dissipative component is maintained as term of the equation.

The electromagnetic theory, as it was presented until now, suggested that the simplest model of the atom of hydrogen could be described as a single plan interaction. Through rendering evident the vector of self-induced interaction, the atomic model becomes a spatial model.

In essence, the model of the hydrogen atom is represented by the electric dipole formed of an electron and a proton. This model could lay at the basis of the atomic description's development or the subsequent development of more complex structures.

The promoted theory opens predictive theoretical perspectives in technology as it proposes a new vision on the structure of substance and the laws that govern it. First of all, it promotes a model in which the determinants are the energy aspects and not the mass quantitative ones.

We are convinced that we have not solved but the beginning of a new way in the problem of the scientific description of the structure of material world, yet we also opened a new way of approach, a closer vision to the general ensemble of the actual scientific knowledge. The theoretical solution of the interaction of two bodies with electric charge is the keystone for the elaboration of a new theoretical model of the structure of material world.

For setting up a new high level theory, the basic hypotheses, respectively the starting points in its elaboration are more important than the formal developments. It should find its foundation in accordance with the other theories or starting from the interpretation of the experimental results. Changing one of the hypotheses, the theory will be changed. These hypotheses should not be brought into accord with the used formalism.

The importance of the new theory is due to precise determination of the position in space of the elementary particles making use of the coordinates related to a non-inertial system of reference and the calculus at any moment of the position with all the variables that describe it.

A big number of the postulates of classical physics have been accepted for admitting the validity of the abstract spatial-temporal structure and for the possibility of pursuing the evolution of the physical world by making use of the well-determined and spatially localized quantities varying continuously in time, and the possibility of describing all the phenomena with the help of a system of differential equations.

In spite of the difficulty of fulfilling the requirements of building such a theory, we succeeded in presenting the organization of hydrogen's structure in accordance with the laws of the classical physics through well-defined measures within the spatial-temporal framework of the classical physics.

The differential equations of the classical mathematical physics have a fundamental feature that allows us to rigorously describe the entire evolution of the analyzed phenomena.

Major differences separate the theory we propose in the epistemological field regarding the **causal** - non causal character of the atomic structure's description, continuous - discontinuous, **determinism** - indeterminism and **certainty** - uncertainty at the knowledge level.

We consider that through our scientific approach we achieved a better theoretical representation, an ampler and more intelligible explicative picture, closer to the common intuition and the classical way of representing the atomic model, generating a beginning in the description and understanding of the phenomena in many scientific fields.

The physics must re-discover and re-interpret the certainties and the classical **determinism** in describing the structure of the atom applying the classical way of thinking to the atomic scale.

This new theory, in my opinion, will become an important scientific instrument for increasing the theoretical understanding of the substance structure, but also a scientific fundament for new technologies through which can be found solutions for the permanently increasing need of energy.

CAPTER 1

ELECTROMAGNETIC CLASSIC PROBLEM OF INTERACTION OF TWO CHARGED PARTICLE IN MOTION

In this chapter is presented an electromagnetic classic problem of interaction of two charged particle in motion witch are interacting by electromagnetic field.

$$ds^2 = g_{ij} dx^i dx^j \tag{1.1}$$

Explicit the matrix Mirkovski:

$$ds^2 = c^2 dt^t - dx^2 - dy^2 - dz^2 \tag{1.2}$$

Results:

$$\begin{aligned} dt' &= dt\sqrt{1-\frac{v^2}{c^2}} \\ v^2 &= \frac{dx^2 + dy^2 + dz^2}{dt^2} \end{aligned} \tag{1.3}$$

From relation (1.3) results the relation between the times of reference systems:

$$t_2' - t_i' = \int_{t_1}^{t_2} dt\sqrt{1-\frac{v^2}{c^2}} \tag{1.4}$$

Action (S) gets the form:

$$S = -mc\int_a^b ds \tag{1.5}$$

The impulse of one particle is defined by the form:

$$\vec{p} = \frac{\partial L}{\partial \vec{v}}\vec{n}_{\vec{v}}$$

or

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1.6}$$

The particle's energy will be:

$$\zeta = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1.7}$$

The lagrengean associated to the particle has the form:

$$L = -mc^2\sqrt{1-\frac{v^2}{c^2}} \tag{1.8}$$

The rest energy will be:

$$\zeta\Big|_{v=0} \cong mc^2 \tag{1.9}$$

Also we have:

$$\zeta\Big|_{v<<0} \cong mc^2 + \frac{mv^2}{2} \tag{1.10}$$

Relation (1.10) can also get the form:

$$\frac{\zeta^2}{c^2} = p^2 + m^2c^2 \tag{1.11}$$

Thus we define the Hamilton function:

$$\begin{gathered} H = c\sqrt{p^2 + m^2c^2} \\ H\Big|_{p<<mc} \cong mc^2 + \frac{p^2}{2m} \end{gathered} \tag{1.12}$$

The impulse of free particle has the form:

$$\vec{p} = \left(\frac{\zeta}{c^2}\right)\vec{v} \tag{1.13}$$

Let's have the variation of action:

$$\delta s = -mc\delta\int_a^b ds = 0 \tag{1.14}$$

which has the form:

$$\delta s = -mcn_i \delta x^i \Big|_b^a + mc\int_b^a \delta\, x^i \frac{dn_i}{ds} ds \tag{1.15}$$

If $\delta\, x^i \big|_a = \delta\, x^i \big|_b = 0$

The result of action's variation will be:

$$\delta s = mc\int_a^b \delta x^i \frac{dn_i}{ds} ds \tag{1.16}$$

The equations of motion are defined by the relation:

$$\delta s = 0 \xrightarrow{become} \frac{dn_i}{ds} = 0 \tag{1.17}$$

To get the variation of action as a co-ordinate function we get a point (A) so that:

$$\delta\, x^i \big|_a = 0 \tag{1.18}$$

(the point (B) is supposed to be variable) so that:

$$\delta\, s\big|_b \equiv \delta\, s = -mcn_i \left(\delta\, x^i\right)_h = -mcn_i \delta\, x^i \tag{1.19}$$

We define the impulse quartet-vector by the form:

$$P_i = -\frac{s}{x^i} \tag{1.20}$$

So that:

$$\begin{aligned} P^i &= mcu^i; \\ P_i &= mcu_i \end{aligned} \tag{1.21}$$

or by the form:

$$\begin{aligned} p_i &= \left(\frac{Z}{c'} - \vec{p} \right) \\ p^i &= \left(\frac{Z}{c'} + \vec{p} \right) \end{aligned} \tag{1.22}$$

it also results the scalar product:

$$p^i p_i = m^2 c^2 u^i u_i = m^2 c^2 \tag{1.23}$$

The force quartet-vector is defined by the form:

$$\begin{aligned} g^i &= \frac{dp^i}{ds} = mc\frac{du^i}{ds} \\ g_i u^i &= mc\frac{du_i}{ds}u^i = 0 \end{aligned} \tag{1.24}$$

If we note:

$$\vec{f} = \frac{d\vec{p}}{dt} \tag{1.25}$$

then the force quartet-vector gets the expression:

$$g^i = \left(\frac{\vec{f}\vec{v}}{c^2\sqrt{1-\frac{v^2}{c^2}}}, \frac{\vec{f}}{c\sqrt{1-\frac{v^2}{c^2}}} \right) \tag{1.26}$$

The Hamilton-Jacobi equation results from the relation:

$$p^i p_i = m^2 c^2 \tag{1.27}$$

and the equation has the form:

$$\frac{1}{c^2}\left(\frac{\partial s}{\partial t}\right)^2 - \left(\frac{\partial s}{\partial x}\right)^2 - \left(\frac{\partial s}{\partial y}\right)^2 - \left(\frac{\partial s}{\partial z}\right)^2 = m^2 c^2 \tag{1.28}$$

Let us define the quartetdimensional kinetic momentum as:

$$\vec{M} = \sum \vec{r} \times \vec{p} \tag{1.29}$$

in quartetdimensional Minkovski system it results:

$$x'^{i} - x^{i} = x_{k}\delta\Omega^{ik}$$
$$x^{i}x^{k}\delta\Omega_{ik} = 0 \qquad (1.30)$$
$$\delta\Omega_{ik} = -\delta\Omega_{ik}$$

The action (S) variation is written starting from the relation:

$$\delta S = \sum p^{i}\delta x_{i}$$

but: $\delta x_{i} = \delta\Omega_{ik}x^{k}$ (1.31)

or by indented the result is:

$$\delta S = \delta\Omega_{ik}\sum p^{i}x^{k}$$

or by identifying the symmetric and anti-symmetric components we will have:

$$\delta S = \delta\Omega_{ik}\frac{1}{2}\sum\left(p^{i}x^{k} - p^{k}x^{i}\right) \qquad (1.32)$$

for an isotropic closed system it becomes:

$$\frac{\partial S}{\partial\Omega_{ik}} = \frac{1}{2}\sum\left(p^{i}x^{k} - p^{k}x^{i}\right) \qquad (1.33)$$

We define:

$$M^{ik} = \sum\left(x^{i}p^{k} - x^{k}p^{i}\right) \qquad (1.34)$$

So we identify:

$$M^{ik} = \left(\sum \left(t \cdot \vec{p} - \frac{\xi \cdot \vec{r}}{c^2} \right) - \vec{M} \right) \tag{1.35}$$

It is conserved for a closed system:

$$\sum \left(t \cdot \vec{p} - \frac{\xi \cdot \vec{r}}{c^2} \right) = const. \tag{1.36}$$

We define the action (S) for a charged particle into an electromagnetic field:

$$S = \int_a^h \left[-mc \cdot ds - \frac{q}{c} \cdot A_i dx^i \right] \tag{1.37}$$

where $A^i = \left[\varphi, \vec{A} \right]$ is the potentially electromagnetic four-vector. The action could be also written as an extended form:

$$S = \int_a^h \left[-mc \cdot \left(\frac{ds}{dt} \right) + \frac{q}{c} \cdot \vec{A} \left(\frac{d\vec{r}}{dt} \right) - q \cdot \varphi \right] \cdot dt \tag{1.38}$$

If:

$$\frac{ds}{dt} = c\sqrt{1 - \frac{v^2}{c^2}} \tag{1.39}$$

then S will get the form:

$$S = \int_a^h \left[-mc^2 \cdot \sqrt{1 - \frac{v^2}{c^2}} + \frac{q}{c} \vec{A} \cdot \vec{V} - q \cdot \varphi \right] \cdot dt \tag{1.40}$$

where: $\vec{A} \rightarrow$ vectorial potential

$\varphi \rightarrow$ scalar potential (electrostatic)

The Lagrange function, for a charged particle in electromagnetic field, written in Mikovski metric gets the form:

$$L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} + \left(\frac{q}{c} \right) \cdot \vec{A} \cdot \vec{V} - q \cdot \varphi \tag{1.41}$$

The Hamilton function, for a charged particle in electromagnetic field gets the form:

$$H = \frac{m \cdot c^2}{\sqrt{1 - \frac{v}{c^2}}} + q \cdot \varphi - q \cdot \vec{V} \cdot \vec{\nabla}_{\vec{V}} \cdot \varphi \tag{1.42}$$

If the particle moving into the field $(\vec{A}, \varphi)$ and do not disturb it, then the Euler-Lagrange equation becomes:

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \vec{V}} \cdot \vec{n}_V \right] - \frac{\partial L}{\partial \vec{r}} \vec{n}_r = 0 \tag{1.43}$$

Explaining the Lagrangian, we get the equations of motion (for the particle do not disturb the field within it is moving):

$$\vec{E} = -\frac{1}{c} \cdot \frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \varphi$$
$$\vec{H} = \vec{\nabla} \times \vec{A}$$
$$\frac{d\vec{P}}{dt} = q \cdot E + \frac{q}{c} \cdot \vec{\nabla} \times \vec{H} \qquad (1.44)$$
$$\vec{P} = \frac{m \cdot \vec{V}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The invariant of rate-seating gets the form:

$$A_k' = A_k - \frac{\partial f}{\partial x^k} \qquad (1.45)$$

Explained by the form:

$$\vec{A}' = A_k - \frac{\partial f}{\partial x^k}$$
$$\varphi' = \varphi - \frac{1}{e} \frac{\partial f}{\partial t} \qquad (1.46)$$

The four vector dimension Euler-Lagrange equations in condition of $\delta S = 0$ get the form:

$$mc\frac{du_i}{ds} = \frac{q}{c}\left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}\right) u^k \tag{1.47}$$

We define the tensor of the electromagnetic field:

$$\vec{t}_{ik} \overset{Def}{=} \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \tag{1.48}$$

The Euler-Lagrange equations as the covariant form are:

$$\begin{aligned} & mc\frac{du_i}{ds} = \frac{q}{c}\vec{F}_{ik} \cdot u^k; \\ & \vec{F}_{ik} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -H_z & H_y \\ -E_y & H_z & 0 & -H_x \\ -E_z & -H_y & H_x & 0 \end{bmatrix} \\ & F_{ik} = \left(\vec{E};\vec{H}\right) \end{aligned} \tag{1.49}$$

or as the contra variance:

$$mc\frac{du_i}{ds} = \frac{q}{c}\vec{F}_{ik} \cdot u^k;$$

$$\vec{F}_{ik} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -H_z & H_y \\ E_y & H_z & 0 & -H_x \\ E_z & -H_y & H_x & 0 \end{bmatrix} \quad (1.50)$$

$$F_{ik} = \left(-\vec{E};\vec{H}\right)$$

The electromagnetic field invariants related to an inertial referential get the form:

$$F_{ik} \cdot F^{ik} = inv \to \vec{H}^2 - \vec{E}^2 = inv \quad (1.51)$$

$$c^{iklm} F_{ik} F_{lm} = inv \to \vec{E} \cdot \vec{H} = inv$$

where:

$$c^{iklm} \cdot c_{prst} = \begin{bmatrix} \delta_p^i & \delta_r^i & \delta_s^i & \delta_t^i \\ \delta_p^k & \delta_r^k & \delta_s^k & \delta_t^k \\ \delta_p^l & \delta_r^l & \delta_s^l & \delta_t^l \\ \delta_p^m & \delta_r^m & \delta_s^m & \delta_t^m \end{bmatrix} \quad (1.52)$$

So that the Maxwell equations on four-vector dimension tensor form become:

$$\frac{\partial F_{ik}}{\partial x^e} + \frac{\partial F_{ke}}{\partial x^i} + \frac{\partial F_{ei}}{\partial x^k} = 0 \quad (1.53)$$

or in a compact form:

$$c^{ikem}\frac{\partial F_{em}}{\partial x^{k}} = 0 \tag{1.54}$$

As follows we analyze the evolution of one system (electromagnetic field – charged particle) as an interaction field-particle.

The action S associated to this system gets the form:

$$S = S_f + S_m + S_{mf} \tag{1.55}$$

We explain the main components of the action:

S_f is that part of action not depending on the field properties (S_f is the action of the field in the absence of the particles).

We present some equivalent expression for S_f:

$$S_f = a \cdot \int\iiint F_{ik}F^{ik} \cdot dv \cdot dt;$$

$$dv = dx \cdot dy \cdot dz$$

$$S_f = -\frac{1}{16\pi \cdot c}\int F_{ik}F^{ik} d\Omega; \tag{1.56}$$

$$d\Omega = c \cdot dt \cdot dxdydz$$

$$S_f = \frac{1}{8\pi}\int\iiint\left[E^2 - H^2\right]dv \cdot dt$$

In conditions of the pure Lagragian field (in the absence of charged particles) has the form:

$$L_f \overset{Def}{=} \frac{1}{8\pi} \int \left[E^2 - H^2\right] dv \cdot dt \tag{1.57}$$

From (1.56) we chose for S_f the expression:

$$S_f = -\frac{1}{16\pi \cdot c} \iiiint F_{ik} F^{ik} d\Omega; \\ d\Omega = c \cdot dt \cdot dxdydz \tag{1.58}$$

S_m – is the part of action that characterizes the particle system in the absence of the electromagnetic field.

For one particle the action S_m has the form:

$$S_m = -mc \int ds \tag{1.59}$$

and for more particles:

$$S_m = -\sum mc \int ds \tag{1.60}$$

It is the part of action that characterizes the interaction between the electromagnetic field and the charged particles. For more particles S_{mf} has the expression:

$$S_{mf} = -\frac{q}{c} \int A_k dx^k \tag{1.61}$$

In conclusion the whole action of the electromagnetic field system – charged particles has the form:

$$S = -\sum \int mc \cdot ds - \frac{q}{c} \int A_k dx^k - \frac{1}{16\pi \cdot c} \int F_{ik} F^{ik} d\Omega \quad (1.62)$$

Where the term (Σ) is regarding to the number of particles interacting with the electromagnetic field. Tensors A_k, F_{ik} effectively depend on the speed and position of the charged particles.

The charged particles are within a continuous, homogeneous and isotropic environment. They are moving along ¨stream¨ trajectories. The charged particles are supposed to be as a point-size so that the density of electric charges has the expression: (where δ is the Dirac function)

$$\rho = \sum_a q_a \delta(\vec{r} - \vec{r}_a) \quad (1.63)$$

The whole charge:

$$Q = \int \rho \cdot dv \quad (1.64)$$

We could define the conductor “stream” such as:

$$I^i = \rho \frac{dx_i}{dt} = \{c\rho, \vec{I}\} \quad (1.65)$$

The whole charge defined at (1.64) becomes:

$$Q = \frac{1}{c}\int I^i ds_i \tag{1.66}$$

ds_i is the hyper-surface to define the stream density

The system of equations that characterizes the whole field-charged particles has the form:

$$F_{ki} = -F^{ik}$$

$$S = -\sum \int mc \cdot ds - \frac{q}{c}\int A_k dx^k - \frac{1}{16\pi \cdot c}\int F_{ik}F^{ik} d\Omega \tag{1.67}$$

$$\frac{\partial I^i}{\partial X^i} = 0 \;\; (\partial I^i = 0);$$

$$d\Omega = c \cdot dt \cdot dx\, dy\, dz$$

The equations of motion for the whole field-charged particle can also be written as:

$$\partial S = 0$$

$$\partial I^i = 0 \tag{1.68}$$

As follows let us define the equation of energy conservation in case of the motion of a sum of bodies within an electromagnetic field (in the absence of particles reaction on the field) for:

$$\left\{-\sum\int mcds=0\right\} \tag{1.69}$$

Let's have the electromagnetic energy equation of conservation form:

$$\iiint\limits_V \left[\frac{\partial}{\partial t}\left(\frac{\vec{E}^2+\vec{H}^2}{8\pi}\right)+\vec{I}\cdot\vec{E}\right]dV=-\oint\!\!\!\oint\limits_{\sum V}\vec{S}\cdot d\vec{\sigma} \tag{1.70}$$

Let's consider that:

$$\int\limits_V \vec{I}\cdot\vec{E}dv\cong\sum 2\cdot\vec{V}\cdot\vec{E} \tag{1.71}$$

(electrical conductance forms)

From the relation $\vec{V}\cdot(\vec{V}\times\vec{H})=0$ results:

$$\mathrm{dE}_{\mathrm{cin}}/dt=q\cdot\vec{E}\cdot\vec{H};$$

The electromagnetic energy equation of conservation becomes:

$$\frac{d}{dt}\left\{\int\limits_V\left[\frac{\vec{E}^2+\vec{H}^2}{8\pi}\right]dv+\sum\mathrm{E}_{\mathrm{cin}}\right\}=-\oint\limits_{\sum V}\vec{S}\cdot d\vec{\sigma} \tag{1.72}$$

We define the **term of tensor energy-impulse** to express the conservation energy law using a four-vector dimensional measure. Let's consider an electromagnetic field without charged particles.

The action of a physic system is defined as:

$$S = \int \Lambda\left(\rho, \frac{\partial \rho}{\partial x^i}\right) dv \cdot dt = \frac{1}{2}\int \Lambda\left(\rho, \frac{\partial \rho}{\partial x^i}\right) d\Omega \qquad (1.73)$$

Where $\Lambda\left(\rho, \frac{\partial \rho}{\partial x^i}\right)$ is the Lagrangean derivative function that determines the state of the physic system.

In case of the electromagnetic field $q \to A_i$, where A_i is the four-vector potential.

For simplification we will use the expressions

$$q_k \to q \text{ (generalized coordinate)}$$

$$\frac{\partial q_k}{\partial x^i} = q_{,i} \text{ (the derivate of a generalized coordinate)} \qquad (1.74)$$

The Lagrange functions of the physic system have the form of the Lagrange density function. The characterized physic system is closed one if:

$$\frac{\partial \Lambda}{\partial x^i} = 0 \qquad (1.75)$$

(Λ does not depend explicitly on x^i)

The physic system's equations of motion are given by the principle of minimum action defined by the equation:

$$\delta S = 0 \qquad (1.76)$$

The equations of motion thus become the Euler-Lagrange equations under the form:

$$\frac{\partial}{\partial x_i}\left(\frac{\partial \Lambda}{\partial q_{,i}}\right)-\frac{\partial \Lambda}{\partial q}=0 \qquad (1.77)$$

or under the form:

$$\frac{\partial T_i^k}{\partial x^k}=0 \qquad (1.78)$$

where: $T_i^k = q_{,i}\frac{\partial \Lambda}{\partial q_{,i}}-\delta_i^k \Lambda$ (1.79)

In case of more (γ) bodies $q \to q^{(e)}$ (generalized coordinates) we will have:

$$T_i^k=\sum_{(e)} q_{,i}^{(e)}\frac{\partial \Lambda}{\partial q_{,i}^{(e)}}-\delta_k^i \Lambda \qquad (1.80)$$

Let's similarly define the tensor energy-impulse:

$$T^{ik}=q_{,i}\frac{\partial \Lambda}{\partial q_{,k}}-\delta^{ik}\Lambda \qquad (1.81)$$

We could also define the potential four vector impulse (P_i), as:

$$P_i = \frac{1}{c}\int T^{ik} dS_k \tag{1.82}$$

The kinetic momentum tensor can also be expressed as:

$$M^{ik} = \frac{1}{c}\int\left[x^i T^{kl} - x^k T^{ie}\right] dS_e \tag{1.83}$$

One should notice that the conservation of the kinetic momentum depends on the tensor energy – impulse symmetry.

$$\left(T^{ki} = T^{ik}\right). \tag{1.84}$$

The explicit form of the tensor energy – impulse is:

$$T^{ik} = \begin{bmatrix} w & S_x/c & S_y/c & S_z/c \\ S_x/c & \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ S_x/c & \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ S_z/c & \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \tag{1.85}$$

where:

$w = T^{oo} \rightarrow$ the density of the electromagnetic energy; $S_{x,y,z}/c \rightarrow$ the impulse density and

$\sigma_{\alpha\beta} \rightarrow$ is a tensor of α impulse, which crosses through the perpendicular surface on direction β .

The calculation example of energy - impulse tensor of the free electromagnetic field:

The Lagrangean of the free electromagnetic field has the form:

$$\Lambda = -\frac{1}{16\pi} F_{kl} F^{kl}$$
$$q \rightarrow A_k \tag{1.86}$$

it results:

$$q_{,i} = \frac{\partial A_k}{\partial x^i}$$
$$T^{ik} = -\frac{1}{4\pi}\left(\frac{\partial A^l}{\partial x_i}\right) F_l^k + \frac{1}{16\pi} g^{ik} F_{lm} F^{lm} \tag{1.87}$$

or the form: (energy-impulse tensor of the free electromagnetic field)

$$T^{ik} = \frac{1}{4\pi}\left[F^{ie} F_s^k + \frac{1}{4}\delta^{ik} F_{lm} F^{ml} \right] \tag{1.88}$$

with the property:

$$T_r\left(T^{ik}\right) = 0 \tag{1.89}$$

The meanings of the impulse- energy tensor components are:

$$T^{00} \equiv W = \frac{E^2 + H^2}{8\pi}$$
$$T^{0\alpha}\Big|_{\alpha=1,2,3} = \frac{S_\alpha}{c}\Big|_{\alpha=1,2,3} \rightarrow \vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{H} \qquad (1.90)$$

(where S_α $\rightarrow$ are the components of Poynting vector)

$$T^{o\alpha} = T^{\alpha 0}$$
$$\sigma_{\alpha\beta} = \frac{1}{4\pi}\left[-E_\alpha E_\beta - H_\alpha H_\beta + + \frac{1}{2}\delta_{\alpha\beta}\left(E^2 + H^2\right)\right] \qquad (1.91)$$

Let's build up the tensor energy - impulse for the whole: electromagnetic field - charge particles.

Let's define the mass repartitions:

$$\mu = \sum_o m_o \delta(\vec{r} - \vec{r}_o) \qquad (1.92)$$

The density of the Q4vector impulse of particles (for their mechanic parts) will be: $\left(\mu \cdot c \cdot u_i\right)$

The density of the spatial impulse will have the expression: - this density represents the components $\left\{T^{o\alpha} / c\right\}$ of the energy-impulse tensor.

$$P^{o\alpha} = \mu c^2 u^d\Big|_{\alpha=1,2,3} \qquad (1.93)$$

The mass density is the temporal component of the four-vector $\left(\frac{\mu}{c}\right)\frac{dx^k}{dt}$

(the same equation for the charge density)

The tensor energy-impulse of the particles system that not interacts with each other but the field has the form:

$$T^{ik} = \mu c \frac{dx^i}{ds}\frac{dx^k}{dt} = \mu c u^i u^k \left(\frac{ds}{dt}\right)$$

(1.94)

where: $u^i = \frac{\partial x^i}{\partial s}; u^k = \frac{dx^k}{ds}$

Let's define:

$T^{(ch)ik} \rightarrow T_i^{(ch)k} \rightarrow$ the impulse-energy tensor for electro-magnetic field

$T^{(P)ik} \rightarrow T_i^{(P)k} \rightarrow$ the impulse-energy tensor for particles (substance)

So that:

$$\frac{\partial}{\partial x^k}\left[T_i^{(ch)k} + T_i^{(p)k}\right] = 0 \quad (1.95)$$

The energy – impulse tensor of the field will by:

$$T_i^{(ch)k} = \frac{1}{4\pi}\left(-F^{il}F_l^k + \frac{1}{4}\delta^{ik}F_{lm}F^{lm}\right) \quad (1.96)$$

The tensor symmetry is obvious. The kinetic momentum is a conservative one: $T_i^i = 0$

Using the Maxwell equations:

$$\frac{\partial F^{kl}}{\partial x^k} = \left(\frac{4\pi}{c}\right) I^l$$
$$\frac{\partial F_{lm}}{\partial x^i} + \frac{\partial F_{mi}}{\partial x^l} + \frac{\partial F_{il}}{\partial x^m} = 0 \qquad (1.97)$$

We have the equation of motion for the field:

$$\frac{\partial T_i^{(ch)k}}{\partial x^k} = -\frac{1}{c} F_{il} I^l \qquad (1.98)$$

We build up a similar equation for the energy – impulse tensor for particles $\left(T_i^{(ch)k}\right)$ so that:

$$T_i^{(ch)k} = \mu c u_i u^k \frac{ds}{dt} \qquad (1.99)$$

The equation of motion for particles that do not interact with each other:

$$\frac{\partial T_i^{(ch)k}}{\partial x^k} = \mu c \frac{du_i}{dt} \qquad (1.100)$$

In calculating the formula (1.100) we used the mass conservation law as follows:

$$\frac{\partial}{\partial x^k}\left[\mu \frac{dx_k}{dt}\right] = 0 \tag{1.101}$$

Let's have the mass and charge distribution defined as:

$$\begin{aligned} \mu &= \sum_o m_o \delta[\vec{r}(t) - \vec{r}_o(t)] \\ \rho &= \sum_o q_o \delta[\vec{r}(t) - \vec{r}_o(t)] \end{aligned} \tag{1.102}$$

it results the equation of motion for the condition $\left\{ \frac{\mu}{m} = \frac{\rho}{q} \right\}$;

$$\mu c \frac{du_i}{ds} = \left(\frac{\rho}{c}\right) F_{ik} u^k \tag{1.103}$$

This equation of motion can also be written as:

$$\begin{aligned} \mu c \frac{du_i}{dt} &= \frac{1}{c} F_{ik} I^k \\ I^k &= \rho \frac{dx^k}{dt} \to \{\rho \vec{v}, c\rho\} \end{aligned} \tag{1.104}$$

Thus, it results:

$$\frac{\partial T_i^{(p)k}}{\partial x^k} = \frac{1}{c} F_{ik} I^k \tag{1.105}$$

If we use the relations (1.98) and (1.100) within the law of conservation (1.95) we notice that the conservation law for the energy – impulse tensor is correct. So that the law of energy conservation is true if:

The equations of motion are:

$$mc\frac{du_i}{ds} = \left(\frac{q}{c}\right) F_{ik} u^k \tag{1.106}$$

On condition that: $\left(\frac{\mu}{m} = \frac{\rho}{q}\right)$

Using the diagonal elements of the energy – impulse total tensor $T_i^{(ch)i} + T_i^{(p)i}$, we have:

$$T_i^i = T_i^{(ch)i} + T_i^{(p)i} \tag{1.107}$$

But,

$$T_i^{(p)i} = \mu c u_i u^i \frac{ds}{dt} = \mu c \frac{ds}{dt} = \mu c^2 \sqrt{1 - \frac{u^2}{c^2}} \tag{1.108}$$

using for μ the expression:

$$\mu = \sum_o m_o \delta(\vec{r} - \vec{r}_o) \tag{1.109}$$

but $T_i^{(ch)i} = 0$ (for the conservation of the kinetic moment) so that:

$$T_i^i = \sum_o m_o c^2 \sqrt{1 - \frac{v_o^2}{c^2}} \delta(\vec{r} - \vec{r}_o) \tag{1.110}$$

Thus, it results that $T_i^i \geq 0$ where $T_i^i = 0$ only for an electromagnetic field without charged particles.

The energy conservation law for particles – field system (the particles do not interact with each other) is presented as in fig. 1

$$\begin{aligned} &\frac{\partial}{\partial x^k}\left[T_i^{(ch)k} + T_i^{(p)k}\right] = 0 \\ &\frac{\partial T_i^{(ch)k}}{\partial x^k} = -\frac{1}{c} F_{il} I^l \\ &\frac{\partial T_i^{(p)k}}{\partial x^k} = \mu c \frac{du_i}{dt} \end{aligned} \tag{1.111}$$

$mc\dfrac{du_i}{ds} = \left(\dfrac{q}{c}\right) F_{ik} u^k \rightarrow$ the charged particles equation of motion into the electromagnetic field.

The particles do not interact with each other and the field is generated by the independent motion of the charged particles.

As follows, we will use the charged particles equations of motion in electromagnetic field under the form:

$$mc\frac{du_i}{ds} = \left(\frac{q}{c}\right)F_{ik}u^k \qquad (1.112)$$

The equations of the electromagnetic field:

$$\frac{\partial F^{kl}}{\partial x^k} = \frac{4\pi}{c}I^l + \frac{4\pi}{c}I^l_{(nonliniar)} \qquad (1.113)$$

$$\frac{\partial F_{lm}}{\partial x^i} + \frac{\partial F_{mi}}{\partial x^l} + \frac{\partial F_{il}}{\partial x^m} = 0$$

Where (1.114)

$$I^l_{nonliniar} = I^l_{nonliniar}(F_{ik})$$

$$u_i = u_i(F_{ik})$$

Represent the four-vectors that demonstrate the presence of self induction in the classic electro-dynamic theory. The equations (2.48) represent the result of a self induction.

The generalization of Maxwell equations consists of it nonlinearity part:

$$
\begin{aligned}
&F_{ik} = F_{ik}(u^i) \\
&I^l_{nonliniar}\left(F_{ik}(u^i)\right) \neq 0
\end{aligned}
\tag{1.115}
$$

The nonlinear effects of the equations (1.113) affect the equation of the covariant tensors evolution modify the values of the torsion (s)-curve tensor, as well as of the impulse-energy tensor.

CHAPTER 2

BUILDING UP THE EVOLUTION EQUATIONS OF A UNIT FORMED OF TWO CHARGED PARTICLES IN ELECTROMAGNETIC INTERACTION

The accurate interpretation of certain experiments regarding the electrodynamics of bodies in motion (Rowland, Eichenwald, Wilson-Wilson, Fizeau, Sagnac-Harres, Mishelson and Morle) required the elaboration of a coherent theory of electrodynamics motion.

Albert Einstein, in his book “Zur Electrodinamik bewergter Körper”, published in 1905, elaborated the theory of restricted relativity, which emphasizes the relative character of the concepts of space, time, simultaneity - admitted as absolute in classical physics.

As a general form they are presented by us in the relations (2.21) and (2.22). Eleven years before Einstein’s work was published, Heinrich Rudolph Hertz in “Uher die Grund-beichungen der Electrodinamik für bewegte Körper” (1890), laid the foundations of a theory of bodies in electrodynamics motion, related to Maxwell’s equations. This theory of bodies in electrodynamics motion was called the Maxwell-Hertz theory. The theory could not explain a series of electromagnetic phenomena taking place at high speed (comparable to the light speed), so it remained tributary to the Galilean kinematics.

The intrinsic model applied by Maxwell for resting mediums and by Hertz for motion mediums was replaced in Einstein works by a kinematic model. Thus, he enounced the two principles that lie at the basis of restricted relativity theory:

- the principle of restricted relativity (the principle of nature laws covariance)
- the principle of the light speed constancy in vacuum.
- the generalization of the vector potential (retarded potentials) $\vec{A}(r_{1,2}, t_{1,2})$ and of the scalar potential $V(r_{1,2}, t_{1,2})$ in non-inertial systems of reference
- the non-linearity of motion equations inducted by the accelerated motion of the two aggregates (which could be conglomerates of particles), similar with Bose-Einstein conglomerates
- the general solution of the system (2.1) is given by the equation $\vec{r}_1 = \vec{r}_1(t_1); \vec{r}_2 = \vec{r}_2(t_2)$

Interpreting the solution of the non-linear system as a coordinates transformation (non-linear due to the field of interaction), we could define a multitude of non-linear solutions for the field of speed ($\vec{V}_1$, $\vec{V}_2$) and energy ($\vec{V}_1^{\,2}+ \vec{V}_2^{\,2}$).

The generalized form of the interaction field is given by the relations (2.7) and (2.8), which define the non-linear system (2.9).

As follows, we will build up the equations of **electromagnetic field and substance interaction** (EFSI). (2.17). Regarding the EFSI process as a process of electromagnetic field's spreading on an inducted "potential", we will use "technique" specific to the "inverse method" of the spreading theory, namely the derivation from the characteristic dimensions of D'Alembert general solution of wave equation.

Let us have the wave equation of EFSI (in Gauss system of measure):

$$\left(\frac{n}{c}\right)^2 \frac{\partial^2 E(t,z)}{\partial t^2} - \frac{\partial^2 E(t,z)}{\partial z^2} = -\frac{n}{c}\cdot\frac{\partial}{\partial t}\left[\alpha \cdot E(t,z) + \frac{4\pi}{nc}\frac{\partial P^{NL}(t,z)}{\partial t}\right] \tag{2.1}$$

The Cauchy restricted problem (initial conditions) is defined as:

$$E(t,z)\big|_{t=0} = \varphi(z) = 0$$

$$\left.\frac{\partial E(t,z)}{\partial t}\right|_{t=0} = \psi(z) = 0 \tag{2.2}$$

In case of regular distribution, the D'Alambert general solution of equation (2.1) within the initial conditions has the form:

$$E(t,z) = \frac{c}{2n}\int_0^t d\tau \int_{z-\frac{c}{n}(t-\tau)}^{z+\frac{c}{n}(t-\tau)} F(\xi,\tau)\,d\xi \tag{2.3}$$

where:

$$F(z,t) = -\frac{n}{c}\frac{\partial}{\partial t}\left[\alpha \cdot E(z,t) + \frac{4\pi}{n \cdot c}\frac{\partial P^{NL}(z,t)}{\partial t}\right] \qquad (2.4)$$

is the non homogeneous term of wave equation that contains, by nonlinear polarization (P^{NL}), the nonlinear terms of EFSI.

Using the coordinate transformation:

$$\begin{aligned} \xi_L &= \frac{c}{n} \cdot t + z \\ \xi_S &= \frac{c}{n} \cdot t - z \end{aligned} \qquad (2.5)$$

The integral equation (2.3) has the form:

$$E(\xi_L, \xi_S) = \frac{c}{2 \cdot n} \int_0^{\frac{n}{2c}(\xi_L + \xi_S)} d\tau \int_{-\xi_S + \frac{c}{n}\tau}^{\xi_L - \frac{c}{n}\tau} F(\xi, \tau) d\xi \qquad (2.6)$$

The partial derivates of the scalar field $E(\xi_L, \xi_S)$ are the derivates by the characteristic directions of waves equation that are presented by the system of coordinate transformation (2.5)

Thus, we have:

$$\frac{\partial E(\xi_L,\xi_S)}{\partial \xi_L}=\frac{c}{2n}\int_0^{\frac{n}{2c}(\xi_L+\xi_S)} F\left[\left(\xi_L-\frac{c}{n}\tau\right),\tau\right]d\tau$$

$$\frac{\partial E(\xi_L,\xi_S)}{\partial \xi_S}=\frac{c}{2n}\int_0^{\frac{n}{2c}(\xi_L+\xi_S)} F\left[\left(-\xi_S+\frac{c}{n}\tau\right),\tau\right]d\tau \qquad (2.7)$$

We explain F(z,t) as:

$$F(z,t)=-\frac{n}{c}\cdot\frac{\partial}{\partial t}g(z,t) \qquad (2.8)$$

where:

$$g(z,t)=\alpha\cdot E(z,t)+\frac{4\pi}{nc}\cdot\frac{\partial P^{NL}(z,t)}{\partial t} \qquad (2.9)$$

- Using relations (2.8) and (2.9) we explain the first integral (2.7) as:

- Let as have the coordinate transformation (that defines "the motion" on direction ξ_L):

$$z = \xi_L - \frac{c}{n}\tau$$
$$t = \tau \tag{2.10}$$

From (2.10) it results:

$$\frac{\partial}{\partial t} = \frac{\partial \tau}{\partial t} \cdot \frac{\partial}{\partial \tau} + \frac{\partial \xi_L}{\partial t} \cdot \frac{\partial}{\partial \xi_L} \tag{2.11}$$

$$F\left[\left(\xi_L - \frac{c}{n}\tau\right), \tau\right] = -\frac{n}{c}\frac{\partial}{\partial \tau} g\left[\left(\xi_L - \frac{c}{n}\tau\right), \tau\right]_{\xi_S = const} \tag{2.12}$$

Thus, the first integral equation from (2.7) has the form:

$$\frac{\partial E}{\partial \xi_L} = -\frac{1}{2}\left[g\left[\frac{1}{2}(\xi_L - \xi_S), \frac{n}{2c}(\xi_L + \xi_S)\right] - g[\xi_L, 0]\right] \tag{2.13}$$

From the condition (2.2) results:

$$g[\xi_L, 0] = 0 \tag{2.14}$$

so the equation (2.13) has the form:

$$\frac{\partial E}{\partial \xi_L} = -\frac{1}{2}\left\{ g\left[\frac{1}{2}(\xi_L - \xi_S), \frac{n}{2c}(\xi_L + \xi_S) \right] \right\}_{\xi_S = const.} \tag{2.15}$$

The equation (2.15) defines “the motion” on the characteristic (ξ_L), where (ξ_S) is a constant value (parameter). In this condition the equation (2.15) has the form:

$$\frac{\partial E}{\partial \xi_L} = -\frac{1}{2} g\left(\frac{\xi_L}{2}, \frac{n}{c}\frac{\xi_L}{2} \right) \tag{2.16}$$

Explaining the function (g), we have, for the component (E_L) of the field, the equation:

$$\frac{\partial E_L}{\partial \xi_L} = -\frac{\alpha}{4} E_L - \frac{\pi}{n^2} \frac{\partial}{\partial \xi_L} P^{NL}(E_L, E_S) \tag{2.17}$$

We analyze now the second integral equation from (2.7). In this case, we have the coordinate transformation:

$$z = -\xi_S + \frac{c}{n}\tau \qquad t = \tau \tag{2.18}$$

$$F\left[\left(-\xi_S+\frac{c}{n}\tau\right),\tau\right]=-\frac{n}{c}\frac{\partial}{\partial\tau}g\left[\left(-\xi_S+\frac{c}{n}\tau\right),\tau\right]_{\xi_L=const} \qquad (2.19)$$

So that, the second integral equation has the form:

$$\frac{\partial E}{\partial\xi_S}=-\frac{1}{2}\left\{g\left[\frac{1}{2}(\xi_L-\xi_S),\frac{n}{2c}(\xi_L+\xi_S)\right]-g[0,\xi_S]\right\} \qquad (2.20)$$

Similar to (2.14), from the condition (2.2) we have the result:

$$g[0,\xi_S]=0 \qquad (2.21)$$

and then the equation (2.20) becomes:

$$\frac{\partial E}{\partial\xi_S}=-\frac{1}{2}\left\{g\left[\frac{1}{2}(\xi_L-\xi_S),\frac{n}{2c}(\xi_L+\xi_S)\right]\right\}_{\xi_L=const.} \qquad (2.22)$$

The equation (2.22) defines "the motion" on the characteristic (ξ_S), where (ξ_L) is a constant value (parameter). In this condition, the equation (2.22) has the form:

$$\frac{\partial E}{\partial\xi_S}=-\frac{1}{2}g\left(-\frac{\xi_S}{2},\frac{n}{c}\frac{\xi_S}{2}\right) \qquad (2.23)$$

Explaining the function (g) we have for the component (E_S) of the field, the equation:

$$\frac{\partial E_S}{\partial \xi_S} = -\frac{\alpha}{4} E_S - \frac{\pi}{n^2} \frac{\partial}{\partial \xi_S} P^{NL}\left(E_L, E_S\right) \tag{2.24}$$

Thus, the equations (2.17) and (2.24) describe the evolution of the electromagnetic field in EFSI process. The same algorithm can be applied in the equation of the electromagnetic field waves inducted EFSI process.

In conclusion we should mention that this method can be used in wave equation (2.1) by making the transformation of

We used this method because:

- in case of the specified EFSI process, the equations on characteristics form a system of coupled quasi-linear equations that allow the continuation of the analytic investigation of EFSI process.

- the equations on characteristics allow a simple use of the initial condition.

Yet, we mention that the Maxwell equations system, which describes the behavior of the electromagnetic field as equations of first order. We will describe these equations using the variables on characteristic curves. Making these systems of equations compact, we will reach an identical result to the one attained by the derivation of D' Alembert general solution.

The geometry of the unit is presented in figure 2.1

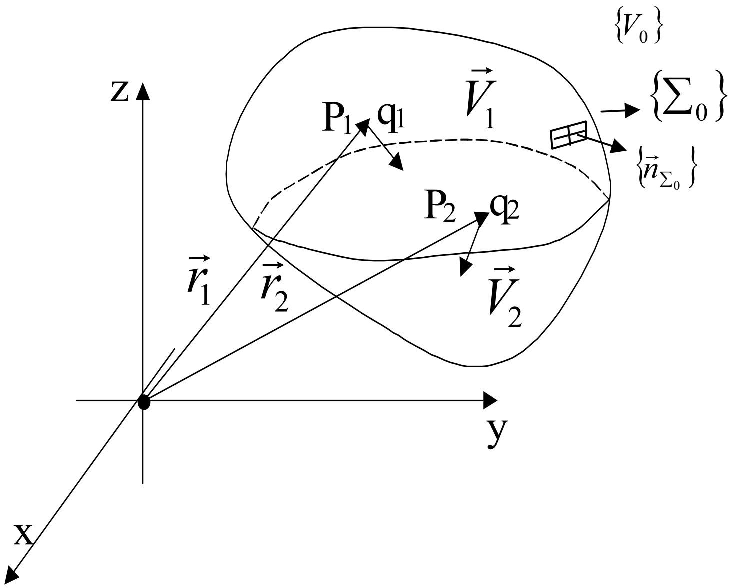


Fig. 2.1

The units consist of two charge points (P_1, P_2) with $\left(\vec{V}_1, \vec{V}_2\right)$ speeds and with the charges (q_1, q_2), being in pure electromagnetic interaction inside of the finite volume (V_0) limited by a finite closed surface (ε_o).

$$\frac{d\vec{r}_1(t_1)}{dt_1} = \vec{V}_1(\vec{r}_1, t_1)$$

$$\frac{d\vec{r}_2(t_2)}{dt_2} = \vec{V}_2(\vec{r}_2, t_2)$$

$$m_1 \cdot \frac{d\vec{V}_1(\vec{r}_1, t_1)}{dt_1} = q\left[-\vec{\nabla}_1 V(\vec{r}_1, t_1) - \frac{\partial \vec{A}(\vec{r}_1, t_1)}{\partial t_1} + V1a \times V1b\right] \tag{2.26}$$

where V1a=$\vec{V}_1(\vec{r}_1, t_1)$ and V1b=$\left\{\vec{\nabla}_1 \times \vec{A}(\vec{r}_1, t_1)\right\}$

$$m_2 \cdot \frac{d\vec{V}_2(\vec{r}_2, t_2)}{dt_2} = q\left[-\vec{\nabla}_2 V(\vec{r}_2, t_2) - \frac{\partial \vec{A}(\vec{r}_2, t_2)}{\partial t_2} + V2a \times V2b\right]$$

Where V2a=$\vec{V}_2(\vec{r}_2, t_2)$ and V2b=$\left\{\vec{\nabla}_2 \times \vec{A}(\vec{r}_2, t_2)\right\}$

$$\vec{A}(\vec{r}_{1,2}, t_{1,2}) = \frac{v}{2\pi^2} \int\int_{-\infty}^{+\infty}\int dk^3 \cdot e^{i\vec{k}\cdot\vec{r}_{1,2}} \cdot$$
$$\cdot \int_{V_0} d\vec{r}\,' e^{-i\vec{k}\cdot\vec{r}\,'} \cdot \int_0^{+\infty} dt' \cdot \frac{\sin kv(t_{1,2} - t')}{k} \cdot \frac{\rho(\vec{r}\,', t')}{\varepsilon} \tag{2.27}$$

where:

$$\vec{\nabla}_1 = \vec{L}\frac{\partial}{\partial x_1} + \vec{J}\frac{\partial}{\partial y_1} + \vec{k}\frac{\partial}{\partial z_1};$$
$$\vec{\nabla}_2 = \vec{L}\frac{\partial}{\partial x_2} + \vec{J}\frac{\partial}{\partial y_2} + \vec{k}\frac{\partial}{\partial z_2} \tag{2.28}$$

Let us explain the quantities $\{\rho, \vec{J}\}$ as:

$$\rho = (\vec{r}\,', t') = \frac{1}{V_0} \cdot$$
$$\cdot \left[q_1 \cdot \delta(t' - t) \cdot \delta(\vec{r}\,' - \vec{r}_1) + q_2 \cdot \delta(t' - t_2) \cdot \delta(\vec{r}\,' - \vec{r}_2)\right] \tag{2.29}$$

Using relations (2.27) we explain the retarded potentials from (2.26) in condition $q_1 = -q_2 = q$; $q = 1{,}6 \cdot 10^{-19}\,C$ .as:

$$V(\vec{r}, t) = B_0\left[f_1(\vec{r}, t; \vec{r}_1, t_1) - f_2(\vec{r}, t; \vec{r}_2, t_2)\right]$$
$$\vec{A}(\vec{r}, t) = A_0\left[\frac{d\vec{r}_1}{dt_1} \cdot f_1(\vec{r}, t; \vec{r}_1, t_1) - \frac{d\vec{r}_2}{dt_2} \cdot f_2(\vec{r}, t; \vec{r}_2, t_2)\right] \tag{2.30}$$

where:

$$B_0 = \frac{c}{2\pi^2} \cdot \frac{q}{\varepsilon_0 \cdot V_0} \tag{2.31}$$

And also where:

$$f_1(\vec{r},t;\vec{r}_1,t_1) = \int\int_{-\infty}^{+\infty}\int \frac{\sin kc(t-t_1)}{k} \cdot e^{i\vec{k}(\vec{r}-\vec{r}_1)} \cdot dk^3$$

$$f_2(\vec{r},t;\vec{r}_2,t_2) = \int\int_{-\infty}^{+\infty}\int \frac{\sin kc(t-t_2)}{k} \cdot e^{i\vec{k}(\vec{r}-\vec{r}_2)} \cdot dk^3 \qquad (2.32)$$

Using relations (2.28) the equations could be written as:

$$\vec{\nabla}V(\vec{r},t) = B_0\left[\vec{\nabla}f_1 - \vec{\nabla}f_2\right]$$

$$\vec{\nabla}\times\vec{A}(\vec{r},t) = A_0\left[\frac{d\vec{r}_2}{dt_2}\times\vec{\nabla}f_2 - \frac{d\vec{r}_1}{dt_1}\times\vec{\nabla}f_1\right] \qquad (2.33)$$

$$\frac{\partial\vec{A}}{\partial t} = A_0\left[\frac{d\vec{r}_1}{dt_1}\cdot\frac{\partial f_1}{\partial t} - \frac{d\vec{r}_2}{dt_2}\cdot\frac{\partial f_2}{\partial t}\right]$$

Using relations (2.32) the expression of the electric field intensity could be written as:

$$\vec{E}(\vec{r},t) = B_0\left[\vec{\nabla}f_2 - \vec{\nabla}f_1\right] + A_0\left[\frac{d\vec{r}_2}{dt_2}\cdot\frac{\partial f_2}{\partial t} - \frac{d\vec{r}_1}{dt_1}\cdot\frac{\partial f_1}{\partial t}\right] \qquad (2.34)$$

and similarly the magnetic induction:

$$\vec{B}(\vec{r},t) = A_0 \cdot \left[\frac{d\vec{r}_2}{dt_2} \times \vec{\nabla} f_2 - \frac{d\vec{r}_1}{dt_1} \times \vec{\nabla} f_1 \right] \tag{2.35}$$

Writing the expression of the electric field intensity $\{\vec{E}\}$ and the magnetic induction $\{\vec{B}\}$ using coordinates $\{\vec{r}_1, t_1\}$ and $\{\vec{r}_2, t_2\}$ as "action" of the field on both particles, we have the equations:

$$\begin{aligned}
& m_1 \cdot \frac{d\vec{V}_1}{dt_1} + \left(A_1 \cdot \gamma_1^*\right) \frac{d\vec{V}_2}{dt_2} = \\
& \left(A_1 c \gamma_2^* + A_1 \left(\vec{V}_1 \cdot \vec{\delta}_1^*\right)\right) \vec{V}_2 + \\
& + \left(B_1 - A_1 \left(\vec{V}_1 \cdot \vec{V}_2\right)\right) \vec{\delta}_1^*
\end{aligned} \tag{2.36}$$

$$\begin{aligned}
& \left(A_1 \gamma_1\right) \cdot \frac{d\vec{V}_1}{dt_1} + m_2 \cdot \frac{d\vec{V}_2}{dt_2} = \\
& \left(- A_1 c \gamma_2 + A_1 \left(\vec{V}_1 \cdot \vec{\delta}_1\right)\right) \vec{V}_1 \\
& - \left(B_1 - A_1 \left(\vec{V}_1 \cdot \vec{V}_2\right)\right) \vec{\delta}_1
\end{aligned}$$

Where “c” is the speed of light in vacuum.

$$A_1 = \frac{c}{2\pi^2} \cdot \mu_0 \cdot \frac{q^2}{V_0}$$

$$B_1 = \frac{c}{2\pi^2} \cdot \frac{1}{\varepsilon_0} \cdot \frac{q^2}{V_0} \tag{2.37}$$

$$\vec{\delta}_1(\vec{r}_1, t_1; \vec{r}_2, t_2) = \\ = \int\int_{-\infty}^{+\infty}\int \frac{i\vec{k}}{k} \cdot \sin kc(t_2 - t_1) e^{i\vec{k}(\vec{r}_2 - \vec{r}_1)} dk^3$$

$$\gamma_1(\vec{r}_1, t_1; \vec{r}_2, t_2) = \\ = \int\int_{-\infty}^{+\infty}\int \frac{1}{k} \cdot \sin kc(t_2 - t_1) e^{i\vec{k}(\vec{r}_2 - \vec{r}_1)} dk^3 \tag{2.38}$$

$$\gamma_2(\vec{r}_1, t_1; \vec{r}_2, t_2) = \\ = \int\int_{-\infty}^{+\infty}\int \cos kc(t_2 - t_1) e^{i\vec{k}(\vec{r}_2 - \vec{r}_1)} dk^3$$

$\delta_1^*, \gamma_1^*, \gamma_2^* \rightarrow$ are the complex - conjugated quantities,

$\delta_1, \gamma_1, \gamma_2, V_0 \rightarrow$ respectively the volume of interaction.

Regarding the last expression one should mention that we assign to $\{V_0\}$ the meaning of "interaction volume", which contains all the oscillation modes of the field – particle interaction.

The equations system (2.35) represents the compact form of (2.27) on hypothesis (2.28).

$$\frac{d\vec{r}_1}{dt_1} = \vec{V}_1$$
$$\frac{d\vec{r}_2}{dt_2} = \vec{V}_2 \tag{2.39}$$

The equations system (2.35) can also be written as:

$$\frac{d\vec{V}_1}{dt_1} = \omega_{11} \cdot \vec{V}_1 + \omega_{12}\vec{V}_2 + A_{2c} \cdot \vec{\delta}_1 + \vec{A}_c \cdot \vec{\delta}_1$$
$$\frac{d\vec{V}_2}{dt_2} = \omega_{21} \cdot \vec{V}_1 + \omega_{22}\vec{V}_2 - A_{1c} \cdot \vec{\delta}_1 + A_c \cdot \vec{\delta}_1^{\,*} \tag{2.40}$$

The gauge relation for generated field of the first particle has the form:

$$\vec{V}_1\vec{\delta}_1 = \frac{B_1}{A_1 c} \cdot \left(\frac{1}{V_0} - \gamma_2 \right) \tag{2.41}$$

and the gauge relation for generated field of the second particle has the form:

$$\vec{V}_2\vec{\delta}_1 = \frac{B_1}{A_1 c}\cdot\left(\frac{1}{V_0} - \gamma_2\right) \tag{2.42}$$

Let us explain the quantities of the motion equations system (2.40) as:

$$\begin{aligned}
\omega_{11} &= \omega_{110} + \Gamma_{110}\cdot\left(\vec{V}_2\cdot\vec{\delta}_1\right)\\
\omega_{12} &= \omega_{120} + \Gamma_{120}\cdot\left(\vec{V}_2\cdot\vec{\delta}_1^{\,*}\right)\\
\omega_{21} &= \omega_{210} - \Gamma_{210}\cdot\left(\vec{V}_2\cdot\vec{\delta}_1\right)\\
\omega_{22} &= -\omega_{220} + \Gamma_{220}\cdot\left(\vec{V}_2\cdot\vec{\delta}_1^{\,*}\right)\\
A_{2c} &= A_{2c_0} - \Gamma_{2c_0}\cdot\left(\vec{V}_1\cdot\vec{V}_2\right)\\
A_{1c} &= A_{1c_0} - \Gamma_{1c_0}\cdot\left(\vec{V}_1\cdot\vec{V}_2\right)\\
A_{c} &= A_{c_0} - \Gamma_{c_0}\cdot\left(\vec{V}_1\cdot\vec{V}_2\right)
\end{aligned} \tag{2.43}$$

As follows in (2.42) we explain the quantities that define "the linear dynamics" of the system as:

$$\omega_{110} = A_1 c \cdot \frac{A_1 \gamma_1^* \gamma_2 - \dfrac{m_2}{V_0}}{m_1 \cdot m_2 - A_1^2 \left|\gamma_1\right|^2}$$

$$\omega_{120} = A_1 c \cdot \frac{m_2 \gamma_2^* - \dfrac{A_1}{V_0} \gamma_1^*}{m_1 \cdot m_2 - A_1^2 \left|\gamma_1\right|^2}$$

$$\omega_{210} = A_1 c \cdot \frac{\dfrac{A_1}{V_0} \gamma_1 - m_1 \gamma_2}{m_1 \cdot m_2 - A_1^2 \left|\gamma_1\right|^2} \tag{2.44}$$

$$\omega_{220} = A_1 c \cdot \frac{A_1 \gamma_1 \gamma_2^* - \dfrac{m_1}{V_0}}{m_1 \cdot m_2 - A_1^2 \left|\gamma_1\right|^2}$$

$$A_{2c_0} = \frac{m_2 \cdot B_1}{m_1 \cdot m_2 - A_1^2 \left|\gamma_1\right|^2}$$

$$A_{1c_0} = \frac{m_1 \cdot B_1}{m_1 \cdot m_2 - A_1^2 \left|\gamma_1\right|^2}$$

$$A_{c_0}^* = \frac{A_1 \cdot B_1 \gamma_1^*}{m_1 \cdot m_2 - A_1^2 |\gamma_1|^2}$$

$$A_{c_0} = \frac{A_1 \cdot B_1 \gamma_1}{m_1 \cdot m_2 - A_1^2 |\gamma_1|^2}$$

In relation (2.43) we define the constants of the nonlinear coupling of the dynamic system as:

$$\Gamma_{110} = \Gamma_{c_0}^* = \frac{A_1^2 \cdot \gamma_1^*}{m_1 \cdot m_2 - A_1^2 |\gamma_1|^2}$$

$$\Gamma_{120} = \Gamma_{2c_0} = \frac{m_2 A_1}{m_1 \cdot m_2 - A_1^2 |\gamma_1|^2}$$

$$\Gamma_{210} = \Gamma_{1c_0} = \frac{m_1 A_1}{m_1 \cdot m_2 - A_1^2 |\gamma_1|^2} \qquad (2.45)$$

$$\Gamma_{220} = \Gamma_{c_0} = \frac{A_1^2 \gamma_1}{m_1 \cdot m_2 - A_1^2 |\gamma_1|^2}$$

The equations of evolution for the square speeds $\left\{\vec{V}_1^{\,2}, \vec{V}_2^{\,2}\right\}$ and for the scalar product $\left\{\vec{V}_1 \cdot \vec{V}_2\right\}$ have the form (in condition of: $t_1 = \left(t_2 + \Delta t\right) = t$

$$\frac{d\vec{V}_1^2}{dt} = \theta_{11} \cdot \vec{V}_1^2 + \theta_{12} \cdot \vec{V}_2^2 + \theta_{13}\left(\vec{V}_1 \cdot \vec{V}_2\right) + \theta_{10}$$

$$\frac{d\vec{V}_2^2}{dt} = \theta_{21} \cdot \vec{V}_1^2 + \theta_{22} \cdot \vec{V}_2^2 + \theta_{23}\left(\vec{V}_1 \cdot \vec{V}_2\right) + \theta_{20} \qquad (2.46)$$

$$\frac{d\left(\vec{V}_1 \cdot \vec{V}_2\right)}{dt} = \theta_{31} \cdot \vec{V}_1^2 + \theta_{32} \cdot \vec{V}_2^2 + \theta_{33}\left(\vec{V}_1 \cdot \vec{V}_2\right) + \theta_{30}$$

The above mentioned system (2.45) was written with the approximation:

$$t_1 = t_2 + \Delta t\left(t_1, t_2\right) = t$$

The general form is described as:

$$\vec{r}_2 = \vec{r}_2\left(\vec{r}_1, t_1, \tau\right)$$
$$t_2 = t_2\left(\vec{r}_1, t_1, \tau\right) \qquad (2.47)$$

or under the form:

$$\vec{r}_1 = \vec{r}_1\left(\vec{r}_2, t_2, \theta\right)$$
$$t_1 = t_1\left(\vec{r}_2, t_2, \theta\right) \qquad (2.48)$$

$$\theta_{12} = 0 \qquad (2.49)$$

$$\theta_{13} = 2 \cdot \left[\omega_{120} + \frac{\Gamma_{120} B_1}{A_1 c} \left(\frac{1}{V_0} - \gamma_2^* \right) \right]$$

$$- 2 \cdot \left[\Gamma_{2c_0} \frac{B_1}{A_1 c} \left(\frac{1}{V_0} - \gamma_2^* \right) + \Gamma_{c_0}^* \frac{B_1}{A_1 c} \left(\frac{1}{V_0} - \gamma_2 \right) \right] \qquad (2.50)$$

As follows we developed the model in condition of {Δt = const.} (stationary conditions of the two charged particles in electromagnetic interaction).

e explain the system's coefficients (2.45) as:

$$\theta_{10} = 2 \left[A_{2c_0} \frac{B_1}{A_1 c} \left(\frac{1}{V_0} - \gamma_2^* \right) \right] +$$

$$+ \left[A_{c_0}^* \frac{B_1}{A_1 c} \left(\frac{1}{V_0} - \gamma_2 \right) \right]$$

$$\theta_{23} = \left[2 \cdot \omega_{210} + \frac{\tau_{210} B_1}{A_1 c} \left(\frac{1}{V_0} - \gamma_2 \right) + \right. \qquad (2.51)$$

$$\left. + \tau_{2c_0} \frac{B_1}{A_1 c} \left(\frac{1}{V_0} - \gamma_2 \right) + \tau_{c_0} \frac{B_1}{A_1 c} \left(\frac{1}{V_0} - \gamma_2^* \right) \right]$$

$$\theta_{20} = -2\left[A_{1c_0} \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2\right) + A_{c_0} \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2^*\right)\right]$$

$$\theta_{10} = 2\left[A_{2c_0} \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2^*\right) + A_{c_0}^* \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2\right)\right]$$

$$\theta_{21} = 0$$

$$\theta_{23} = \left[2 \cdot \omega_{210} + \frac{\tau_{210} B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2\right) + \right.$$

$$\left. + \tau_{2c_0} \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2\right) + \tau_{c_0} \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2^*\right)\right] \qquad (2.52)$$

$$\theta_{20} = -2\left[A_{1c_0} \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2\right) + A_{c_0} \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2^*\right)\right]$$

$$\theta_{31} = \omega_{210} - \frac{\Gamma_{210} \cdot B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2\right)$$

$$\theta_{32} = \omega_{120} - \frac{\Gamma_{120} \cdot B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2^*\right)$$

$$\theta_{33} = \left[\omega_{110} - \omega_{220} + \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2\right)\left(\Gamma_{110} + \Gamma_{1c_0} - \Gamma_{c_0}^*\right) + \right.$$

$$\left. + \frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2^*\right) \cdot \left(\Gamma_{220} + \Gamma_{c_0} - \Gamma_{2c_0}\right)\right]$$

$$\theta_{30} = \left[\left(A_{2c_0} - A_{c_0}\right)\frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2^*\right)\right] + \tag{2.53}$$

$$+\left[\left(A_{c_0}^* - A_{1c_0}\right)\frac{B_1}{A_1 c}\left(\frac{1}{V_0} - \gamma_2\right)\right]$$

The form of coefficients $\{\theta_{ij}\}$ is simplified by using the relation between them:

$$\frac{B_1}{A_1 c^2} = 1 \tag{2.54}$$

We continue the analytic investigation in conditions:

$$m_1 \cdot m_2 >> A_1^2 \cdot |\gamma_1|^2 \tag{2.55}$$

In these conditions, the coefficients of motion equations have the form:

$$\omega_{110} = 0;$$

$$\omega_{120} = +\frac{A_1 c \gamma_2^*}{m_1} = +\frac{A_1 c}{m_1} \cdot \tag{2.56}$$

$$\cdot \int\int_{-\infty}^{+\infty}\int \cos kc(t_2 - t_1) e^{-i\vec{k}(\vec{r}_2 - \vec{r}_1)} dk^3$$

$$\omega_{210} = -\frac{A_1 c \gamma_2}{m_2} = -\frac{A_1 c}{m_2} \cdot$$

$$\cdot \int\int_{-\infty}^{+\infty}\int \cos kc(t_2 - t_1) e^{+i\vec{k}(\vec{r}_2 - \vec{r}_1)} dk^3$$

$$\omega_{220} = 0;$$

$$A_{2c_0} = \frac{B_1}{m_1};$$

$$A_{1c_0} = \frac{B_1}{m_2}; \tag{2.57}$$

$$A_{c_0} = 0;$$

$$\Gamma_{110} = \Gamma_{220} = \Gamma_{c0}^* = \Gamma_{c0} \cong 0$$

$$\tau_{120} = \frac{A_1}{m_1} = \tau_{2c0}$$

$$\tau_{210} = \frac{A_1}{m_2} = \tau_{1c0}$$

Thus, the dynamic equations system will have the form:

$$\frac{d\vec{V}_1}{dt_1} = \vec{V}_1 + \left[\frac{A_1 c \gamma_2}{m_1} + \frac{A_1}{m_1} \left(\vec{V}_1 \cdot \vec{\delta}_1 \right) \right] \cdot \vec{V}_2 +$$
$$+ \left[\frac{B_1}{m_1} - \frac{A_1}{m_1} \left(\vec{V}_1 \cdot \vec{V}_2 \right) \right] \cdot \vec{\delta}_1$$

(2.58)

$$\frac{d\vec{V}_2}{dt_2} = \left[-\frac{A_1 c \gamma_2}{m_2} - \frac{A_1}{m_2} \left(\vec{V}_2 \cdot \vec{\delta}_1 \right) \right] \vec{V}_1 +$$
$$+ \cdot \vec{V}_2 - \left[\frac{B_1}{m_2} - \frac{A_1}{m_2} \left(\vec{V}_1 \cdot \vec{V}_2 \right) \right] \cdot \vec{\delta}_1$$

As follows, we rate-set the equations system:

At the beginning we define the constant with the physic meaning of frequency:

$$\omega_0 = \frac{A_1 c}{\sqrt{m_1 \cdot m_2} \cdot V_0} \tag{2.59}$$

where V_0 is the finite volume of interaction

We rate-set the variable of time:

$$\xi_1 = \omega_0 \cdot t_1 \; ; \; \xi_2 = \omega_0 \cdot t_2$$

$$\vec{t}_{ik} \overset{Def}{=} \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \tag{2.60}$$

We rate-set the speeds to the speed of light (c):

$$\vec{V}_1{}' = \frac{\vec{V}_1}{c}$$

$$\vec{V}_2{}' = \frac{\vec{V}_2}{c} \tag{2.61}$$

We set the rate distances:

$$\vec{r}_1{}' = \frac{\vec{r}_1}{r_{01}}$$

$$\vec{r}_2{}' = \frac{\vec{r}_2}{r_{02}} \tag{2.62}$$

where:

$$r_{01} = \frac{c}{\omega_0}\sqrt{\frac{m_1}{m_2}}$$

$$r_{02} = \frac{c}{\omega_0}\sqrt{\frac{m_2}{m_1}} \tag{2.63}$$

We rate-set again the variables of time as:

$$\eta_1 = \omega_0 \sqrt{\frac{m_2}{m_1}} t_1$$

$$\eta_2 = \omega_0 \sqrt{\frac{m_1}{m_2}} t_2 \tag{2.64}$$

The system of equations of rate-set variables has the form:

$$\frac{d\vec{r}_1'}{d\eta_1} = \vec{V}_1'$$

$$\frac{d\vec{r}_2'}{d\eta_2} = \vec{V}_2'$$

$$\frac{d\vec{V}'_1}{d\eta_1} = \left[G_0 + \left(\vec{V}_1' \cdot \vec{D}_1\right)\right] \cdot \vec{V}_2' + \left[1 - \left(\vec{V}_1' \vec{V}_2'\right)\right] \cdot \vec{D}_1 \tag{2.65}$$

$$\frac{d\vec{V}'_2}{d\eta_2} = -\left[G_0 + \left(\vec{V}_2' \cdot \vec{D}_1\right)\right] \cdot \vec{V}_1' - \left[1 - \left(\vec{V}_1' \vec{V}_2'\right)\right] \cdot \vec{D}_1$$

where:

$$G_0 = \gamma_2 \cdot V_0 = V_0 \cdot$$

$$\cdot \int\int\limits_{-\infty}^{+\infty}\int \cos\left|\vec{k}\right| c\left(t_2 - t_1\right) \cdot e^{i\vec{k}(\vec{r}_2 - \vec{r}_1)} d\left|\vec{k}\right|^3$$

$$\vec{D}_1 = V_0 \cdot \vec{\delta}_1 = V_0 \cdot \tag{2.66}$$

$$\cdot \int\int\limits_{-\infty}^{+\infty}\int \frac{i\vec{k}}{\left|\vec{k}\right|} \sin\left|\vec{k}\right| c\left(t_2 - t_1\right) \cdot e^{i\vec{k}(\vec{r}_2 - \vec{r}_1)} d\left|\vec{k}\right|^3$$

where: $d\left|\vec{k}\right|^3 = dk_x dk_y dk_z$; $\left|\vec{k}\right| = k$;

We will express G and $\vec{D}$ in spherical coordinates:

$$k_x = k \cos\theta \sin\varphi$$

$$k_y = k \sin\theta \sin\varphi \tag{2.67}$$

$$k_z = k \cdot \cos\varphi$$

Where the volume element has the form:

$$dk^3 = k^2 \cdot \sin\varphi \cdot dk \cdot d\theta \cdot d\varphi \tag{2.68}$$

We observe that $\{G_0\}$ and $\{\vec{D}_1\}$ verify the homogenous wave equation:

$$\vec{\nabla}^2\left(\vec{r}_2'-\frac{m_1}{m_2}\vec{r}_1'\right)G_0\left(\vec{r}_2'-\frac{m_1}{m_2}\vec{r}_1',\quad \eta_2-\frac{m_1}{m_2}\eta_1\right)-$$
$$=\frac{\partial^2 G_0\left(\vec{r}_2'-\frac{m_1}{m_2}\vec{r}_1',\quad \eta_2-\frac{m_1}{m_2}\eta_1\right)}{\partial\left(\eta_2-\frac{m_1}{m_2}\eta_1\right)^2}=0 \tag{2.69}$$

$$\vec{\nabla}^2\left(\vec{r}_2'-\frac{m_1}{m_2}\vec{r}_1'\right)D_1\left(\vec{r}_2'-\frac{m_1}{m_2}\vec{r}_1',\quad \eta_2-\frac{m_1}{m_2}\eta_1\right)-$$
$$-\frac{\partial^2 \vec{D}_1\left(\vec{r}_2'-\frac{m_1}{m_2}\vec{r}_1',\quad \eta_2-\frac{m_1}{m_2}\eta_1\right)}{\partial\left(\eta_2-\frac{m_1}{m_2}\eta_1\right)^2}=0$$

The particular solutions of the wave equations have the form:

$$\left\{\cos\left[\alpha_1\left(\eta_2-\frac{m_1}{m_2}\eta_1\right)-\alpha_2\left(x_2'-\frac{m_1}{m_2}x_1'\right)-\right.\right.$$
$$\left.\left.-\alpha_3\left(y_2'-\frac{m_1}{m_2}y_1'\right)-\alpha_4\left(z_2'-\frac{m_1}{m_2}z_1'\right)+\varphi\right]\right\}_{\alpha_1^2=\alpha_2^2+\alpha_3^2+\alpha_4^2} \tag{2.70}$$

$$\left\{\sin\left[\alpha_1\left(\eta_2-\frac{m_1}{m_2}\eta_1\right)-\alpha_2\left(x_2'-\frac{m_1}{m_2}x_1'\right)-\right.\right.$$
$$\left.\left.-\alpha_3\left(y_2'-\frac{m_1}{m_2}y_1'\right)-\alpha_4\left(z_2'-\frac{m_1}{m_2}z_1'\right)+\varphi\right]\right\}_{\alpha_1^2=\alpha_2^2+\alpha_3^2+\alpha_4^2}$$

The particular solutions of the wave equations are valid depending on the existence of the relation between the four parameters $\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$:

$$\alpha_1^2=\alpha_2^2+\alpha_3^2+\alpha_4^2 \tag{2.70}$$

There are the pairs of integer or rational numbers $\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$, which comply with the relation (2.70). We attach to every groups of numbers $\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$ the notion of {n} type "oscillation mode" where {n} is natural number.

So that the pairs of numbers $\{\alpha_{i_n}\}_{\substack{i=1,2,3,4\\ n\in N}}$ define a "dynamic state" of the system of two charged particles in electromagnetic interaction.

We have now the gauge relation for $\{G_0\}$ and $\{\vec{D}_1\}$ as:

$$\frac{\partial G_0}{\partial(t_2-t_1)}-c\cdot\vec{\nabla}(\vec{r}_2-\vec{r}_1)\vec{D}=0 \tag{2.71}$$

In -set rate variables, the gauge relation has the form:

$$\frac{\partial G_0\left(\eta_2-\frac{m_1}{m_2}\eta_1;\vec{r}_2'-\frac{m_1}{m_2}\vec{r}_1'\right)}{\partial\left(\eta_2-\frac{m_1}{m_2}\eta_1\right)}-$$
$$-\vec{\nabla}\left(\vec{r}_2'-\frac{m_1}{m_2}\vec{r}_1'\right)\cdot\vec{D}_1\left(\eta_2-\frac{m_1}{m_2}\eta_1;\quad \vec{r}_2'-\frac{m_1}{m_2}\vec{r}_1',\right)=0 \tag{2.72}$$

As follows, we present a model of calculation.

Let us have:

$$\left\{\begin{array}{l} G_0=\sin\left[\begin{array}{l}\alpha_1\left(\eta_2-\frac{m_1}{m_2}\eta_1\right)-\alpha_2\left(x_2'-\frac{m_1}{m_2}x_1'\right)\\ -\alpha_3\left(y_2'-\frac{m_1}{m_2}y_1'\right)-\alpha_4\left(z_2'-\frac{m_1}{m_2}z_1'\right)+\varphi\end{array}\right] \\ \vec{D}_1=\left[\vec{L}\cdot\beta_1+\vec{I}\cdot\beta_2+\vec{k}\beta_3\right]\cdot G_0\end{array}\right\} \tag{2.73}$$

the gauge relation has the form:

$$[\alpha_1+\beta_1\alpha_2+\beta_2\alpha_3+\beta_3\alpha_4]G_0=0 \tag{2.74}$$

In fact the gauge relation is an additional condition for the series of parameters $\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$ defined as:

$$\alpha_1+\beta_1\alpha_2+\beta_2\alpha_3+\beta_3\alpha_4=0 \tag{2.75}$$

In conclusion, the field equations could be written as:

The quantities $\beta_1, \beta_2, \beta_3 \rightarrow$ could be the frequencies rate-set on the three orthogonal directions of the nonlinear curves.

$$\frac{d\vec{r}_1'}{d\eta_1} = \vec{V}_1'$$

$$\frac{d\vec{r}_2'}{d\eta_2} = \vec{V}_2'$$

$$\frac{d\vec{V}_1'}{d\eta_1} = G_0\vec{V}_2' + \left(\vec{V}_1' \cdot \vec{D}_1\right) \cdot \vec{V}_2' - \left(\vec{V}_1' \vec{V}_2'\right) \cdot \vec{D}_1 + \vec{D}_1 \tag{2.76}$$

$$\frac{d\vec{V}_2'}{d\eta_2} = G_0\vec{V}_1' + \left(\vec{V}_2' \cdot \vec{D}_1\right) \cdot \vec{V}_1' + \left(\vec{V}_1' \vec{V}_2'\right) \cdot \vec{D}_1 - \vec{D}_1$$

Therefore, we could write the relations:

$$\vec{V}_1' \times \left(\vec{V}_2' \times \vec{D}_1\right) = \left(\vec{V}_1' \cdot \vec{D}_1\right)\vec{V}_2' - \left(\vec{V}_1' \cdot \vec{V}_2'\right) \cdot \vec{D}_1$$

$$\vec{V}_2' \times \left(\vec{V}_1' \times \vec{D}_1\right) = \left(\vec{V}_2' \cdot \vec{D}_1\right)\vec{V}_1' - \left(\vec{V}_2' \cdot \vec{V}_1'\right) \cdot \vec{D}_1$$

$$G_0 = \sin\left[\alpha_1\left(\eta_2 - \frac{m_1}{m_2}\eta_1\right) - \alpha_2\left(x_2' - \frac{m_1}{m_2}x_1'\right) - \right. \tag{2.77}$$

$$\left. - \alpha_3\left(y_2' - \frac{m_1}{m_2}y_1'\right) - \alpha_4\left(z_2' - \frac{m_1}{m_2}z_1'\right) + \varphi\right]$$

2.1 The interaction generalization of electromagnetic waves and charged electric particle in motion

The classic form of the Maxwell equations

$$\begin{aligned}
&\frac{\partial \vec{E}}{\partial t} = c^2 \cdot \vec{\nabla} \times \vec{B} \\
&\vec{\nabla} \cdot \vec{E} = 0 \\
&\frac{\partial B}{\partial t} = - \cdot \vec{\nabla} \times \vec{E} \\
&\vec{V} \cdot \vec{B} = 0
\end{aligned} \tag{2.78}$$

The Poynting vector:

$$\vec{P} = \vec{E} \times \vec{V} \tag{2.79}$$

The wave equations:

$$\begin{aligned}
&\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial^2 t^2} = 0 \\
&\Delta \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial^2 t^2} = 0
\end{aligned} \tag{2.80}$$

The equations of the dynamic system with self induction could be written as a compact form:

$$\frac{\partial \vec{E}}{\partial t} = c^2 \vec{\nabla} \times \vec{B} - c\left(\vec{\nabla} \cdot \vec{E}\right) \frac{\vec{E} \times \vec{B}}{\left|\vec{E} \times \vec{B}\right|}$$

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} \qquad (2.81)$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\frac{\partial \rho}{\partial t} = -c \cdot \vec{\nabla}\left(\rho \cdot \vec{I}\right)$$

$$\rho = \vec{\nabla} \cdot \vec{E} \qquad (2.82)$$

The conservation equation:

$$\frac{1}{2} \cdot \frac{\partial}{\partial t}\left(\left|\vec{E}\right|^2 + C^2 \left|\vec{B}\right|^2\right) =$$

$$= C^2 \cdot \left(\vec{\nabla} \times \vec{B} \cdot \vec{E} - \vec{\nabla} \times \vec{E} \cdot \vec{B}\right) = -C^2 \vec{\nabla} \vec{P} \qquad (2.83)$$

$$\vec{I} \cdot \vec{I} = 1$$

$$\vec{E} \cdot \vec{I} = 0 \qquad (2.84)$$

$$E \frac{\partial \vec{I}}{\partial t} = \frac{\partial\left(\vec{E} \cdot \vec{I}\right)}{\partial t} - \frac{\partial \vec{E}}{\partial t} \cdot \vec{I} = C^2 \cdot \left(\vec{\nabla} \times \vec{B}\right)\vec{I} + C \cdot \vec{\nabla} \vec{E}$$

This results in:

$$\begin{aligned}&\frac{\partial^2 \vec{B}}{\partial t^2} = -\vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} = \\&= -C^2 \cdot \vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) + C \cdot \vec{\nabla} \times \left(\rho \cdot \vec{I}\right) = \\&= -C^2 \vec{\nabla}\left(\vec{\nabla}\vec{B}\right) + C^2 \Delta \vec{B} + C \cdot \vec{\nabla} \times \left(\rho \cdot \vec{I}\right)\end{aligned} \tag{2.85}$$

The wave equation with nonlinear terms:

$$\frac{\partial^2 \vec{B}}{\partial t^2} = C^2 \Delta \vec{B} + C \cdot \vec{\nabla} \times \left(\rho \cdot \vec{I}\right) \tag{2.86}$$

The movement equations:

$$\begin{aligned}&\frac{d\vec{r}_1}{d\eta} = \vec{V}_1 \\&\frac{d\vec{r}_2}{d\eta} = \vec{V}_2 \\&\frac{d\vec{V}_1}{d\eta_1} = G_0 \cdot \vec{V}_2 + \vec{V}_1 \times \left(\vec{V}_2 \times \vec{D}_1\right) + \vec{D}_1 \\&\frac{d\vec{V}_2}{d\eta_2} = G_0 \cdot \vec{V}_1 + \vec{V}_2 \times \left(\vec{V}_1 \times \vec{D}_1\right) + \vec{D}_1\end{aligned} \tag{2.87}$$

This equations are the **generalization of the Maxwell equations**, which consists in deductively adding a new non-dissipative term within the equation, o reaction component of the interaction field - substance. This active component of electromagnetic nature, which we called **vector of self-induced interaction** of the **structural dipole,** represents the component of distortion of dipole’s interaction, different from zero. The dissipative component is maintained as term of the equation.

$$\vec{B} = \mu \cdot \vec{H}$$
$$\vec{D} = \varepsilon \cdot \vec{E} \tag{2.88}$$

$$\vec{\sigma} = \alpha\left[\left(\vec{E} \times \vec{H}\right) \cdot \vec{\nabla}\right] + \beta\left[\left(\vec{\nabla} \times \vec{E}\right) \cdot \vec{H}\right]$$
$$+ \gamma\left[\left(\vec{H} \times \vec{\nabla}\right) \cdot \vec{E}\right] + \delta\left[\left(\vec{\nabla} \times \vec{H}\right) \cdot \vec{E}\right] \tag{2.89}$$

$$\vec{\sigma} = \alpha \cdot \vec{P} \cdot \vec{\nabla} + \beta \cdot \left(-\frac{\partial \vec{B}}{\partial t} \cdot \vec{H}\right)$$
$$+ \gamma \cdot \left(\vec{H} \times \vec{\nabla}\right) \cdot \vec{E} + \delta \cdot \left(\sigma_{1D} \cdot \vec{E} + \frac{\partial \vec{D}}{\partial t}\right) \cdot \vec{E} \tag{2.90}$$

$$\vec{\sigma} = \alpha \cdot \vec{P} \cdot \vec{\nabla} + \beta \cdot \left(-\frac{\partial \vec{B}}{\partial t} \cdot \vec{H} \right) + \gamma \cdot \left(\vec{H} \times \vec{\nabla} \right) \cdot \vec{E} + \delta \cdot \left(\sigma_{1D} \cdot \vec{E} + \frac{\partial \vec{D}}{\partial t} \right) \cdot \vec{E} \quad (2.90)$$

$$\vec{\sigma}^{2} = \frac{1}{2} \cdot \frac{\partial}{\partial t} \left(\vec{H}^{2} + \vec{E}^{2} \right) + \delta \sigma_{1D} \vec{E}^{2} + \alpha \cdot \vec{P} \cdot \vec{\nabla} + + \gamma \cdot \left(\vec{H} \times \vec{\nabla} \right) \cdot \vec{E} \quad (2.91)$$

The complete Maxwell equations (that describe the temporal evolution) have the following form:

$$\frac{\partial \vec{E}}{\partial t} = C^{2} \left(\vec{\nabla} \times \vec{B} \right) - C \left(\vec{\nabla} \vec{E} \right) \cdot \frac{\vec{E} \times \vec{B}}{\left| \vec{E} \times \vec{B} \right|}$$
$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} - C \left(\vec{\nabla} \vec{B} \right) \cdot \frac{\vec{E} \times \vec{B}}{\left| \vec{E} \times \vec{B} \right|} \quad (2.92)$$

We propose a temporal evolution equation system:

$$\begin{aligned}
&\frac{\partial \vec{E}}{\partial t} = C^2\left(\vec{\nabla} \times \vec{B}\right) - \left(\vec{\nabla}\vec{E}\right)\cdot \vec{V} \\
&\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} - \left(\vec{\nabla}\vec{B}\right)\cdot \vec{V} \\
&\frac{\partial \vec{V}}{\partial t} = -\left(\vec{V} \cdot \vec{V}\right)\cdot \vec{V} + \mu\left(\vec{E} + \vec{V} \times \vec{B}\right) \\
&\left|\vec{V}\right| = C
\end{aligned} \tag{2.93}$$

where μ is a slow dimensional-equivalent measure of the *(q/η_e)* form.

The 2.93 equations will take the form of:

$$\begin{aligned}
&\vec{G} = \frac{\Delta \vec{V}}{\Delta t} = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \vec{V}\right)\cdot \vec{V} \\
&\frac{\Delta \vec{V}}{\Delta t} = \mu\left(\vec{E} + \vec{V} \times \vec{B}\right) \\
&\frac{\Delta \vec{V}}{\Delta t} \rightarrow external \quad derivative
\end{aligned} \tag{2.94}$$

From the 2.94 equations will result:

$$\frac{\Delta\vec{V}}{\Delta t}\cdot\vec{V}=\frac{\partial\vec{V}}{\partial t}\cdot\vec{V}+\left(\frac{1}{2}\vec{\nabla}\left|\vec{V}\right|^{2}-\vec{V}\times\left(\vec{\nabla}\times\vec{V}\right)\right)\vec{V}=0$$
$$\vec{E}\cdot\vec{I}=0 \qquad (2.95)$$
$$\vec{E}\cdot\vec{V}=0$$
$$\vec{B}\cdot\vec{I}=0$$

The conservation equations

$$\frac{1}{2}\cdot\frac{\partial}{\partial t}\left[\left|\vec{E}\right|^{2}+C^{2}\cdot\left|\vec{B}\right|^{2}\right]= \qquad (2.96)$$
$$-C^{2}\vec{\nabla}\left(\vec{E}\times\vec{B}\right)-C^{2}\left(\vec{\nabla}\cdot\vec{B}\right)\left(\vec{\nabla}\cdot\vec{B}\right)$$

where

$$\vec{V}\times\frac{\Delta\vec{V}}{\Delta t}=\mu\left(\vec{V}\times\vec{E}-C^{2}\vec{B}+\left(\vec{V}\cdot\vec{B}\right)\vec{V}\right) \qquad (2.97)$$

CHAPTER 3

NON-STATIONARY REGIME IN THE PROCESS OF INTERACTION OF THREE WAVES, FIELDS OR PARTICLES

The non-stationary regime is defined by the equations that describe the interaction of three particles. The supplementary terms of this system of equations bring new information on the interaction process. of interacting particles.

In this chapter, we will demonstrate that these terms lead to the existence of the singular solutions which describe the movement of the elementary particles in the process of atomic interaction.

Using the methods of determination of the algebraic invariants that characterize the system of autonomous differentials equations described in chapter 2, we find out new properties of the interaction process.

3.1 The interaction equations of three fields as a system of differential non-linear and autonomous equations

Let us have the Maxwell equations with nonlinear polarization $\{\vec{P}_{NL}\}$ in Gaussian system of measurement.

$$\vec{\nabla}\, x\, \vec{B} = \alpha n \vec{E} + \frac{n^2}{c} \cdot \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \cdot \frac{\partial \vec{P}_{N_L}}{\partial t}$$

$$\vec{\nabla}\, x\, \vec{E} = -\frac{1}{c} \cdot \frac{\partial \vec{B}}{\partial t}; \qquad (3.1)$$

$$\vec{\nabla} \cdot \vec{E} = 0;$$

$$\vec{\nabla} \cdot \vec{B} = 0;$$

where c is the speed of light in vacuum

n is the coefficient of refraction of the interaction medium

$\vec{E}$ is the electric field vector;

$\vec{B}$ is the magnetic induction vector;

α is the coefficient of attenuation of electromagnetic energy within the interaction medium.

$$\vec{\nabla} = \vec{I} \frac{\partial}{\partial X} + \vec{J} \frac{\partial}{\partial y} + \vec{K} \frac{\partial}{\partial z'}$$

According to the geometry of the interaction process we will have the following expressions for the variables of the Maxwell equations:

$$\vec{E} = E_x \vec{I} + E_y \vec{J}$$

$$\vec{B} = B_x \vec{I} + B_y \vec{J} \tag{3.2}$$

$$\vec{P}_{NL} = P_{NLx} \vec{I} + P_{NLy} \vec{J}$$

The intensity of the electric $\vec{E}$ and magnetic $\vec{B}$ field are described by a vectorial system with cylindrical geometry that excludes the Z component.

By introducing the 3.2 equations to the 3.1 equations, the Maxwell equations become:

$$\frac{n}{c}\cdot\frac{\partial E_x}{\partial t}+\frac{1}{n}\cdot\frac{\partial B_y}{\partial z'}=-\alpha E_x-\frac{4\pi}{c\cdot n}\cdot\frac{\partial P_{NL\,x}}{\partial t};$$

$$\frac{n}{c}\cdot\frac{\partial E_y}{\partial t}-\frac{1}{n}\cdot\frac{\partial B_x}{\partial z'}=-\alpha E_y-\frac{4\pi}{c\cdot n}\cdot\frac{\partial P_{NL\,y}}{\partial t}$$

$$\frac{\partial E_x}{\partial z'}+\frac{n}{c}\cdot\frac{\partial}{\partial t}\left(\frac{B_y}{n}\right)=0 \qquad (3.3)$$

$$\frac{\partial E_y}{\partial z'}-\frac{n}{c}\cdot\frac{\partial}{\partial t}\left(\frac{B_x}{n}\right)=0$$

$$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=0$$

$$\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=0$$

We now take into account the expression of the characteristic equations in space – time description

$$\xi_L=\frac{c}{n}t+z'$$

$$\xi_S=\frac{c}{n}t-z' \qquad (3.4)$$

where ξ_L and ξ_S represent two field phases defined in the (t,z) space.

ξ_L and ξ_S define the space and time characteristics for a compact description of the electromagnetic field components.

The operational relations associated with these characteristics have the form:

$$\begin{aligned}
&\frac{n}{c}\cdot\frac{\partial}{\partial t}+\frac{\partial}{\partial z'}=2\cdot\frac{\partial}{\partial \xi_L};\\
&\frac{n}{c}\cdot\frac{\partial}{\partial t}-\frac{\partial}{\partial z'}=2\cdot\frac{\partial}{\partial \xi_S};\\
&\frac{n}{c}\cdot\frac{\partial}{\partial t}=\frac{\partial}{\partial \xi_S}+\frac{\partial}{\partial \xi_L};\\
&\frac{\partial}{\partial z'}=\frac{\partial}{\partial \xi_L}-\frac{\partial}{\partial \xi_S};
\end{aligned} \tag{3.5}$$

This is how the operators to the 3.4 characteristic equations are defined.

Using these relations we express the Maxwell equations with the new variables $\{\xi_L, \xi_S\}$ and obtain:

$$\frac{\partial}{\partial \xi_L}\left(E_x+\frac{B_y}{n}\right)-\frac{\partial}{\partial \xi_S}\left(E_x-\frac{B_y}{n}\right)=0 \tag{3.6}$$

Is the characteristic linear equation for the B_y field component.

In the case of movement on the characteristic $\{\Gamma_1\}$, we define the initial condition:

$$\frac{\partial}{\partial \xi_L}\left(E_x + \frac{B_y}{n}\right) - \frac{\partial}{\partial \xi_S}\left(E_x - \frac{B_y}{n}\right) = 0$$

$$\left(\frac{dz'}{dt}\right)_{\varphi_S} = -\left(\frac{d(z_c^{'} - z^{'})}{dt}\right)_{\varphi_S} \qquad (3.7)$$

ξ_L defines the slope of the incident field phase

ξ_S defines the deflected field phase

ξ_F is the result of the two phases ξ_L and ξ_S

$$\frac{\partial}{\partial \xi_L}\left(E_x + \frac{B_y}{n}\right) = -\frac{\alpha}{2}E_x - \frac{2\pi}{n^2}\left(\frac{\partial}{\partial \xi_L} + \frac{\partial}{\partial \xi_S}\right)P_{NLx} \qquad (3.8)$$

P_{NLx} - is the nonlinear polarized state of the propagation media for which:

$$\frac{\partial}{\partial \xi_S}\left(E_x - \frac{B_y}{n}\right) = 0 \qquad (3.9)$$

The last two equations represent the evolution equations for the electric field intensities B_x and B_y

$$\frac{\partial}{\partial \xi_S}\left(E_x - \frac{B_y}{n}\right) = -\frac{\alpha}{2}E_x - \frac{2\pi}{n^2}\left(\frac{\partial}{\partial \xi_L} + \frac{\partial}{\partial \xi_S}\right)P_{NLx} \qquad (3.10)$$

for which:

$$\frac{\partial}{\partial \xi_L}\left(E_x + \frac{B_y}{n}\right) = 0 \qquad (3.11)$$

So we have the result:

$$\frac{\partial}{\partial \xi_L}\left(E_y - \frac{B_x}{n}\right) = -\frac{\alpha}{2}E_y - \frac{2\pi}{n^2}\left(\frac{\partial}{\partial \xi_L} + \frac{\partial}{\partial \xi_S}\right)P_{NLy} \qquad (3.12)$$

for which:

$$\frac{\partial}{\partial \xi_S}\left(E_y + \frac{B_x}{n}\right) = 0 \qquad (3.13)$$

Then subtract from (3.32) with (3.34) and we have:

$$\frac{\partial}{\partial \xi_S}\left(E_y + \frac{B_x}{n}\right) = -\frac{\alpha}{2}E_y - \frac{2\pi}{n^2}\left(\frac{\partial}{\partial \xi_L} + \frac{\partial}{\partial \xi_S}\right)P_{NLy} \qquad (3.14)$$

for which:

$$\frac{\partial}{\partial \xi_L}\left(E_y - \frac{B_x}{n}\right) = 0 \qquad (3.15)$$

we obtain the following expression for the inducted electromagnetic field associated to the characteristic $\{\xi_L\}$:

$$\frac{\partial E_L}{\partial \xi_L} = -\frac{\alpha}{4}E_L - \frac{\pi}{n^2} \cdot \frac{\partial}{\partial \xi_L} P_{NL}(E_{L,S}) \qquad (3.16)$$

This equation defines the evolution of the E_L field component on the ξ_L characteristic

And we also obtain the following expression for the radiant electromagnetic field associated to the characteristic $\{\xi_S\}$:

Similar for ξ_S :

$$\frac{\partial E_S}{\partial \xi_S} = -\frac{\alpha}{4} E_S - \frac{\pi}{n^2} \cdot \frac{\partial}{\partial \xi_S} P_{NL}(E_{L,S}) \qquad (3.17)$$

Where:

E_L is the amplitude of the induction electromagnetic field

E_S is the amplitude of the radiant electromagnetic field

In these equations we express the nonlinear polarization as a function of the perturbation of the normal density of the interaction medium, and obtain:

$$\frac{\partial E_L}{\partial \xi_L} = -\frac{\alpha}{4} E_L - \frac{\pi \cdot \gamma^e}{n^2} \cdot \frac{\partial}{\partial \xi_L} (E_S \cdot \frac{\Delta\rho}{\rho_0});$$

$$(3.18)$$

$$\frac{\partial E_S}{\partial \xi_S} = -\frac{\alpha}{4} E_S + \frac{\pi \cdot \gamma^e}{n^2} \cdot \frac{\partial}{\partial \xi_S} (E_L \cdot \frac{\Delta\rho}{\rho_0});$$

where $E_L(\xi_L, \xi_S)$ is the amplitude of the induction field.

$E_S(\xi_L, \xi_S)$ is the amplitude of the EFSI attenuation field.

$\frac{\Delta\rho(\xi_L,\xi_S)}{\rho_0}$ is the perturbation of the normal density of the interaction medium

The evolution equations of the electromagnetic field ξ_L and ξ_S are described in Chapter 2 using the integrated d'Alambert equations under generalized form

$$\frac{\partial}{\partial t}\left(\frac{\Delta\rho}{\rho_0}\right)+\frac{\partial V}{\partial z'}=0;V=\frac{dz'}{dt}$$

$$\frac{v^2}{\gamma}\cdot\frac{\partial}{\partial z'}\left(\frac{\Delta\rho}{\rho_0}\right)+\frac{\partial V}{\partial t}= \tag{3.19}$$
$$=\frac{\partial}{\partial z'}\left[\left(\frac{\eta_B}{\rho_0}\right)\frac{\partial V}{\partial z'}+\frac{\gamma^e}{8\pi\rho_0}(E_L+E_S)^2\right]$$

The hydrodynamic equations (Navier – Stokes type) have been written in hypothesis:

v - is the speed of sound within the propagation media;

γ - is the adiabatic coefficient.

Let us use the next relations:

$$\frac{v^2}{\gamma}=\frac{\omega^2}{K^2};$$
$$\frac{\eta_B}{\rho_0}=\frac{\Gamma_B}{K^2}; \tag{3.20}$$

where $\Gamma_B = \frac{1}{\tau}$ is the length of the spectral line for the electromagnetic field.

If we use these relations in the Maxwell equations we obtain:

$$\frac{\partial}{\partial t}\left(\frac{\Delta\rho}{\rho_0}\right) + \frac{\partial V}{\partial z'} = 0; V = \frac{dz'}{dt}$$

$$\omega^2 \cdot \frac{\partial}{\partial z'}\left(\frac{\Delta\rho}{\rho_0}\right) + K^2 \frac{\partial V}{\partial t} = \qquad (3.21)$$

$$= \frac{\partial}{\partial z'}\left[(\Gamma_B)\frac{\partial V}{\partial z'} + \frac{\gamma^e K^2}{8\pi\rho_0}(E_L + E_S)^2\right]$$

This equation describes the isentropic compression of the nonlinear electromagnetic propagation field.

The characteristic equations for the electromagnetic field have the form:

$$\xi_{1f} = \frac{\omega}{K}t + z'$$

$$\xi_{2f} = \frac{\omega}{K}t - z' \qquad (3.22)$$

ξ_{1f} and ξ_{2f} are phases of the compression field and also characteristic equations for this type of field.

With the new variables $\{\xi_{1f}, \xi_{2f}\}$ the equations have the form $\{\Delta\rho / \rho_0 = \rho'\}$;

$$\frac{\partial}{\partial\xi_{1f}}(\rho' + \frac{K}{\omega}V) + \frac{\partial}{\partial\xi_{2f}}(\rho' - \frac{K}{\omega}V) = 0$$

$$\frac{\partial}{\partial\xi_{1f}}(\rho' + \frac{K}{\omega}V) - \frac{\partial}{\partial\xi_{2f}}(\rho' - \frac{K}{\omega}V) = \left(\frac{\partial}{\partial\xi_{1f}} - \frac{\partial}{\partial\xi_{2f}}\right)\cdot \qquad (3.23)$$

$$\cdot\left[-\frac{\Gamma_B}{\omega K}\left(\frac{\partial}{\partial\xi_{1f}} + \frac{\partial}{\partial\xi_{2f}}\right)\rho' + \frac{\gamma^e \cdot K^2}{8\pi\rho_0\omega^2}(E_L + E_S)^2\right]$$

Adding the two expressions it results:

$$2\cdot\frac{\partial}{\partial\xi_{1f}}(\rho' + \frac{K}{\omega}V) = \left(\frac{\partial}{\partial\xi_{1f}} - \frac{\partial}{\partial\xi_{2f}}\right)\cdot$$

$$\cdot\left[-\frac{\Gamma_B}{\omega K}\left(\frac{\partial}{\partial\xi_{1f}} + \frac{\partial}{\partial\xi_{2f}}\right)\rho' + \frac{\gamma^e \cdot K^2}{8\pi\rho_0\omega^2}(E_L + E_S)^2\right]; \qquad (3.24)$$

for which:

$$\frac{\partial}{\partial\xi_{2f}}(\rho' - \frac{K}{\omega}V) = 0 \qquad (3.25)$$

Then subtract the two expressions from (3.23) and results:

$$2\cdot\frac{\partial}{\partial\xi_{2f}}(\rho'-\frac{K}{\omega}V)=-\left(\frac{\partial}{\partial\xi_{1f}}-\frac{\partial}{\partial\xi_{2f}}\right)\cdot \cdot\left[-\frac{\Gamma_B}{\omega K}\left(\frac{\partial}{\partial\xi_{1f}}+\frac{\partial}{\partial\xi_{2f}}\right)\rho'+\frac{\gamma^e\cdot K^2}{8\pi\rho_0\omega^2}(E_L+E_S)^2\right] \tag{3.26}$$

for which:

$$\frac{\partial}{\partial\xi_{1f}}(\rho'+\frac{K}{\omega}V)=0 \tag{3.27}$$

The equations of evolution on the two characteristics $\{\xi_{1f}, \xi_{2f}\}$ of measure $\{\rho'=\Delta\rho/\rho_0\}$ have the form:

We observe that the two equations are identical as form and we will use the prime integral on characteristic $\{\xi_{1f}\}$ taking into account the line Stokes selection, respectively:

are evolution equations for the fields that are interacting based on the characteristic equations.

3.2 PARTICULAR INTERACTION CASES

For the equations described before, we search for solutions in the form of:

$$x' = x_1 \cdot e^{i\varphi_L} + x_1^* \cdot e^{-i\varphi_L}$$
$$y' = y_1 \cdot e^{i\varphi_S} + y_1^* \cdot e^{-i\varphi_S} \quad (3.31)$$
$$z = z_1 \cdot e^{i\varphi_f} + z_1^* \cdot e^{-i\varphi_f}$$

where $\{x^*_1\}$ represents the complex conjugated measure of $\{x_1\}$ $\left(i = \sqrt{-1}\right)$.

Using as well the complex conjugated equations of the system of equations (3.31), we have a system of quasi-linear hyperbolic equations of evolution.

Using the relations of transformation (3.35), the equations of evolution (3.30) have the form:

$$\frac{\partial}{\partial \varphi_f}(x_1 + y_1 \cdot z_1) + i\left(\delta \frac{\omega_L}{\omega}\right)(x_1 + y_1 \cdot z_1) =$$
$$= -\frac{\alpha'}{2}\left(\delta \frac{\omega_L}{\omega}\right) \cdot x_1$$
$$\frac{\partial}{\partial \varphi_f}(y_1 - x_1 \cdot z_1^{*}) + i\left(\delta \frac{\omega_L}{\omega}\right)(y_1 - x_1 \cdot z_1^{*}) \qquad (3.35)$$
$$= -\frac{\alpha'}{2} \cdot \left(\delta \frac{\omega_L}{\omega}\right) y_1$$
$$\frac{\partial}{\partial \varphi_f} z_1 + i_1 \cdot z_1 = -(2A) z_1 + \sigma_1 \cdot x_1 \cdot y_1^{*}$$

The particular equations for the normal amplitudes of the electric field component (x_1) of the reflected electric field component (y_1) and of the isentropic field component (z_1)

As follows, we will use nonlinear algebraic transformations for having the system (3.35) as a quasi-linear hyperbolic differential equations system, calculating the Riemann invariants:

For the beginning let's have the transformation:

$$\begin{aligned} x_1 + y_1 \cdot z_1 &= f_1 \\ -z_1^{*} x_1 + y_1 &= f_2 \end{aligned} \qquad (3.36)$$

Let us have the evolution equations for the implicit function (real) of the Riemann invariants type:

$$x'' = f_1 \cdot f_1^*$$
$$y'' = f_2 \cdot f_2^*$$
$$W = z_1 \cdot z_1^* \tag{3.37}$$
$$V = z_1 \cdot f_1^* \cdot f_2 + z_1^* \cdot f_1 \cdot f_2^*$$

Using the relation (3.36) and (3.37) will build up the equation of evolution for the function $\{z_1 f_1^* \cdot f_2\}$.

$$\frac{\partial(z_1 f_1^* \cdot f_2)}{\partial \varphi_f} =$$
$$= -\left[(2A) + \frac{\alpha'}{2}\delta\left(\frac{2\omega_L}{\omega}\right)\cdot\frac{1}{1+z_1 z_1^*}\right]\cdot z_1 \cdot f_1^* \cdot f +$$
$$+\frac{\alpha'}{2}\left(\delta\frac{\omega_L}{\omega}\right)\cdot\frac{z_1 z_1^* \cdot f_2 f_2^*}{1+z_1 z_1^*} - \frac{\alpha'}{2}\left(\delta\frac{\omega_L}{\omega}\right)\cdot\frac{z_1 z_1^* \cdot f_1 f_1^*}{1+z_1 z_1^*} + \sigma \tag{3.38}$$
$$\cdot\frac{f_1 f_1^* \cdot f_2 f_2^* + (z_1 f_1^* f_2)(f_1 f_1^*)}{(1+z_1 z_1^*)^2} -$$
$$-\frac{(z_1 f_1^* f_2)(f_2 f_2^*) - (z_1 f_1^* \cdot f_2)^2}{(1+z_1 z_1^*)^2}$$

The right part of the equations (3.36) are real and so the function ($z_1 f_1^* \cdot f_2$) is real too. In these conditions, from relation (3.38) results:

$$V = 2 z_1 f_1^* \cdot f_2 \tag{3.39}$$

or

$$V^2 = 4 \cdot W \cdot x'' \cdot y''; \tag{3.40}$$

Using the linear transformation, we will have a simpler form:

$$\begin{aligned} x'' + y'' &= x_2 \\ x'' - y'' &= y_2 \end{aligned} \tag{3.41}$$

and the propriety:

$$W \langle\langle 1; \tag{3.42}$$

In these conditions, the system of hyperbolic quasi-linear equations has the form:

$$\begin{aligned}
&\frac{\partial x_2}{\partial \varphi_f} = -\alpha'\left(\delta \frac{\omega_L}{\omega}\right) \cdot x_2; \\
&\frac{\partial y_2}{\partial \varphi_f} = -\alpha'\left(\delta \frac{\omega_L}{\omega}\right) \cdot (y_2 - V); \\
&\frac{\partial V}{\partial \varphi_f} = \\
&= -\left(2A + \alpha'\delta \frac{\omega_L}{\omega}\right) \cdot V - \left(\alpha'\delta \frac{\omega_L}{\omega}\right) \cdot W \cdot y_2 + \\
&+ \frac{\sigma}{2}(x_2^2 - y_2^2) + \sigma \cdot V \cdot y_2 - 2\sigma \cdot V^2; \\
&\frac{\partial W}{\partial \varphi_f} = -(4A) \cdot W + \sigma \cdot V - \sigma \cdot V \cdot W + 2\sigma \cdot W \cdot y_2;
\end{aligned} \tag{3.43}$$

The Cauchy for the system (3.43) could have the form:

$$
\begin{aligned}
& x_2(\varphi_f)\Big|_{\varphi_f=\varphi_{f0}} = x_{20}(\varphi_{f_0}); \\
& y_2(\varphi_f)\Big|_{\varphi_f=\varphi_{f0}} = y_{20}(\varphi_{f_0}); \\
& V(\varphi_f)\Big|_{\varphi_f=\varphi_{f0}} = V_0(\varphi_{f_0}); \\
& W(\varphi_f)\Big|_{\varphi_f=\varphi_{f0}} = W_0(\varphi_{f_0});
\end{aligned}
\qquad (3.44)
$$

3.3. THE STUDY OF THE ANALYTIC SOLUTIONS FOR A THREE-WAVE INTERACTION

The system of equations completely describes the process of three-wave interaction. We calculate the analytic solutions of this system for energy loss conditions

So that this is an essential condition for the loss of energy, given that representing the envelop of electromagnetic field inducted in nonlinear medium.

Thus, the system is reduced to a system of two equations, respectively:

$$\frac{\partial V}{\partial \eta} = -V^2 + \frac{1}{2}\left(y_{20} - \frac{2A}{\sigma_1}\right)\cdot V + \left(\frac{x_{20}^2 - y_{20}^2}{4}\right)$$
$$\frac{\partial W}{\partial \eta} = \left(y_{20} - \frac{2A}{\sigma_1}\right)\cdot W + \frac{1}{2}V - \frac{1}{2}V\cdot W \tag{3.45}$$

$\left(L_B \to L'_B \Rightarrow \sigma \to \sigma_1 \right)$

where:

$$\eta = 2\sigma_1\varphi_f \tag{3.46}$$

If we introduce the Cauchy functions as:

$$V_0 = \frac{1}{4}\left(y_{20} - \frac{2A}{\sigma_1} \right).$$
$$\Delta_0^2 = V_0^2 + \frac{1}{4}(x_{20}^2 - y_{20}^2) \qquad (3.47)$$

where:

$$x_1 x_1^*(\eta)\Big|_{\varphi_f = \varphi_{f0}(Z'=0)} = x_{10}^{'}(\eta_0);$$
$$y_1 y_1^*(\eta)\Big|_{\varphi_f = \varphi_{f0}(Z'=0)} = y_{10}^{'}(\eta_0)$$
$$V(\eta)\Big|_{\varphi_f = \varphi_{f0}(Z'=0)} = V_{10}(\eta_0) \qquad (3.48)$$
$$\eta(\varphi_f)\Big|_{\varphi_f = \varphi_{f0}(Z'=0)} = \eta_0(\varphi_{f_0});$$

$x_1 x_1^*(\eta)$ - is the prime integral on characteristic $\{\Gamma_2\}$

$y_1 y_1^*(\eta)$ - is the prime integral on characteristic $\{\Gamma_1$

$$x_1 x_1^*(\eta) = x_{10}(\eta_0) + (2 \cdot W - W_0) y_{10} + V_{10} - V$$
$$y_1 y_1^*(\eta) = (2 \cdot W - W_0) \cdot x_{10} + y_{10} - V_{10} + V \qquad (3.49)$$

The equations represent intensity values for the electro-magnetic field under the conditions of the existence of the V_0 , Δ_0 constants.

In conclusion, in a non-stationary regime, the maximum compression ratio is higher than in the quasi-stationary one. Finally, we should mention the influence of the extinction coefficient of the **spontaneous induction** upon the compression ratio: by increasing the spontaneous induction intensity, the maximum value of the compression ratio decreases, both in quasi stationary and non stationary regime.

As follows, we analyze the case of α‘ ≠ 0. In this case, the Riemann invariants will be: {V_0 (η), Δ_0^2 (η), V(η), W (η) } where the functions {V_0 (η), Δ_0^2 (η)} are defined similar as in the relation (3.25), the difference consisting in the fact that, in this case, these functions depend on the functions {V(η), W(η)} as defined in relation (3.20). Following the algebraic transformation of the system defined in (3.19) results an equation system under the form:

$$
\begin{aligned}
&\frac{\partial V_0}{\partial \eta} = -(4\gamma_1)\cdot V_0 + (\gamma_1)\cdot V - (\gamma_2) \\
&\frac{\partial \Delta_0^2}{\partial \eta} = -(2\gamma_2)\cdot V_0 - (2\gamma_2)\cdot V - \\
&-(8\gamma_1)\cdot \Delta_0^2 - (6\gamma_1) V_0 \cdot V \\
&\frac{\partial V}{\partial \eta} = \Delta_0^2 - V^2 - V_0^2 + 2\cdot V\cdot V_0 - \\
&-(16\gamma_1) W \cdot V_0 - (4\gamma_2) W \\
&\frac{\partial W}{\partial \eta} = (8\gamma_1) W + \left(\frac{1}{2}\right)\cdot V + 4\cdot V_0 \cdot W
\end{aligned}
\qquad (3.53)
$$

where:

$$\gamma_1 = \frac{\alpha'}{8\sigma_1}\left(\delta \cdot \frac{\omega_L}{\omega}\right)$$
$$\gamma_2 = \frac{\alpha'}{8\sigma_1}\left(\delta \cdot \frac{\omega_L}{\omega}\right)\left(\frac{A}{\sigma_1} + \frac{\alpha'}{\sigma_1}\delta \cdot \frac{\omega_L}{\omega}\right) \tag{3.54}$$

with the properties:

$$\lim_{\alpha' \to 0} \gamma_1 = 0$$
$$\lim_{\alpha' \to 0} \gamma_2 = 0 \tag{3.55}$$
$$\lim_{\alpha' \to 0} \frac{\gamma_2}{\gamma_1} = \frac{A}{\sigma_1}$$

We attach to the system (3.57) the implicit equation (3.15), which depend on the new invariants, has the form:

$$V^2 = 4 \cdot W \cdot (\Delta_0^2 - V_0^2) \tag{3.56}$$

Using the implicit relation (3.60), we reduce the system (3.57) to three equations using the Riemann invariants:

$$N_1(\eta) \equiv V_0(\eta)$$
$$N_2(\eta) = \sqrt{\Delta_0^2 - V_0^2} \tag{3.57}$$
$$N_3(\eta) = \sqrt{W}$$

The system of equations (3.57) has the form:

$$\frac{\partial N_1}{\partial \eta} = -(4\gamma_1)\cdot N_1 + (2\gamma_1)\cdot N_2 N_3 - (\gamma_2)$$

$$\frac{\partial N_2}{\partial \eta} = -(4\gamma_1)\cdot N_2 - (2\gamma_2)\cdot N_3 - (8\gamma_1)\cdot N_1 N_3 \tag{3.58}$$

$$\frac{\partial N_3}{\partial \eta} = +(4\gamma_1) N_3 + \left(\frac{1}{2}\right)\cdot N_2 +$$

$$+(2)\cdot N_1 N_3 - (2)\cdot N_2 N_3^2$$

Let's have the invariants {N1(η), N2(η), N3(η)} as function of { $x_1 x_1^*$;· $y_1 y_1^*$; $W(\eta) = z_1 z_1^*(\eta)$ } and we get the relations:

$$x_1 x_1^* - y_1 y_1^* = \frac{4(1-N_3^2)}{1+3N_3^2}\left(N_1 + \frac{\gamma_2}{4\gamma_1} - \frac{N_2\cdot N_3}{1-N_3^2}\right)$$

$$(x_1 x_1^*)\cdot(y_1 y_1^*) =$$

$$= \left[\frac{N_2}{1-N_3^2} + \frac{4N_3}{1+3N_3^2}\left(N_1 + \frac{\gamma_2}{4\gamma_1} - \frac{N_2\cdot N_3}{1-N_3^2}\right)\right]^2 \tag{3.59}$$

$$z_1\cdot z_1^* = N_3^2$$

These measures are components of the algebraic invariants.

If:

$$x_1 x_1^* - y_1 y_1^* = \varphi_1$$
$$(x_1 x_1^*) \cdot (y_1 y_1^*) = \varphi_2 \tag{3.60}$$

Then results:

$$x_1 x_1^* = \frac{2\varphi_2}{\sqrt{\varphi_1^2 + 4\varphi_2} - \varphi_1}$$
$$y_1 y_1^* = \frac{2\varphi_2}{\sqrt{\varphi_1^2 + 4\varphi_2} + \varphi_1} \tag{3.61}$$

Using the components of the algebraic invariants we obtain solutions for the field intensities in a much more exact form than that of the approximate form of the nonlinear equations.

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